Simulating few- and many-body physics with Rydberg atoms

David Petrosyan

Plan of the Seminar

- Quantum Simulations and Computations

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- Rydberg atoms and their interactions

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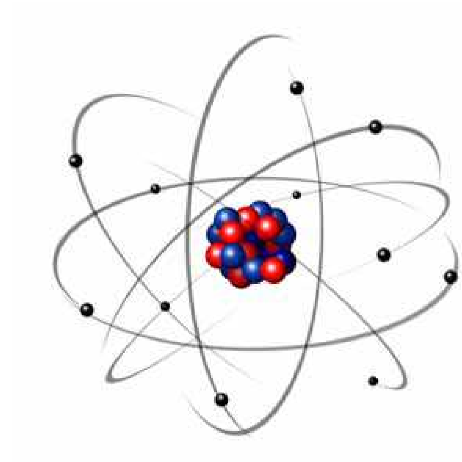
- Summary



Outline of Quantum Theory

Quantum state in a Hilbert Space

$|\Psi\rangle$ is a vector in a complex vector space $\mathbb{H}$

$$|\Psi\rangle = c_1 |e_1\rangle + c_2 |e_2\rangle + c_3 |e_3\rangle + \ldots = \sum_{i=1}^{N} c_i |e_i\rangle \qquad \sum_{i=1}^{N} |c_i|^2 = 1$$

basis $\{ |e_i\rangle\}$: eigenstates of some operator $\mathcal{O} |e_i\rangle = O_i |e_i\rangle$

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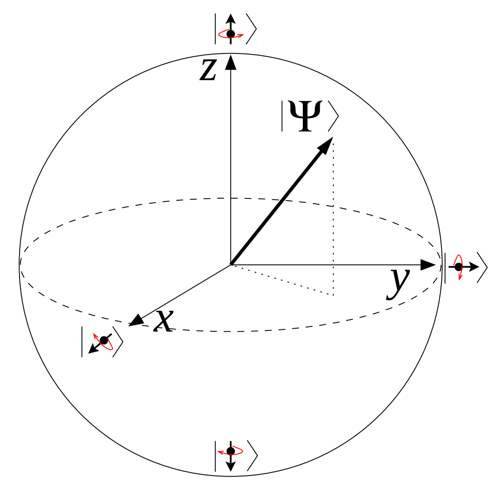
Two state system: spin-$\frac{1}{2}$ electron $\{ |\uparrow\rangle, |\downarrow\rangle\}$



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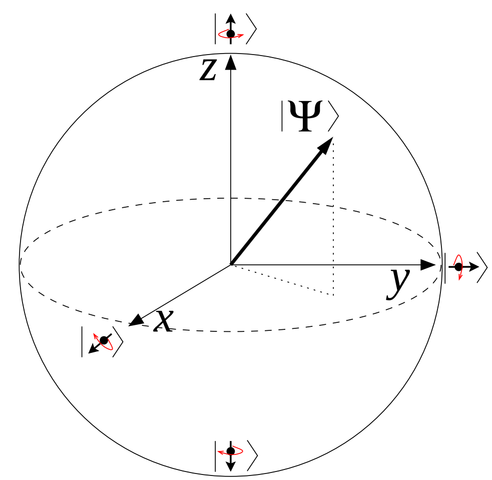

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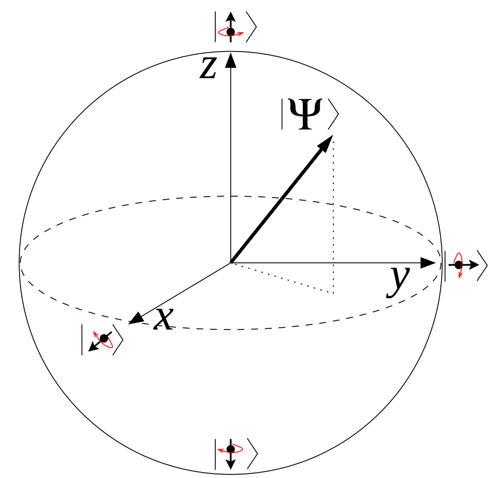

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Hilbert Space is a big place!

(Carlton M. Caves)

Time evolution

Hamiltonian operator $\mathcal{H}$: Energy $\langle\Psi|\,\mathcal{H}\,|\Psi\rangle = E$

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Schrödinger equation $\boxed{i\hbar\frac{\partial}{\partial t}\,|\Psi\rangle = \mathcal{H}\,|\Psi\rangle}$ $\Rightarrow$ $|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\mathcal{H}t}\,|\Psi(0)\rangle$

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Mixed states, dissipative systems: $\hat{\rho} = \sum_{\Psi} P_{\Psi}\,|\Psi\rangle\langle\Psi|$

Liouville – von Neumann equation $\boxed{i\hbar\frac{\partial}{\partial t}\hat{\rho} = [\mathcal{H}\hat{\rho} - \hat{\rho}\mathcal{H}] + \mathcal{L}\hat{\rho}}$

Quantum simulations

Simulating Physics with Computers,

Feynman, Int. J. Theor. Phys. **21**, 467 (1982);
Lloyd, Science **273**, 1073 (1996)

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Interacting many-body quantum systems are hard to simulate on classical computers

- Many degrees of freedom (huge $\mathbb{H}_S$)
- Quantum correlations (entanglement) $|\Psi\rangle_S \neq |\Psi\rangle_A \otimes |\Psi\rangle_B \otimes \dots$

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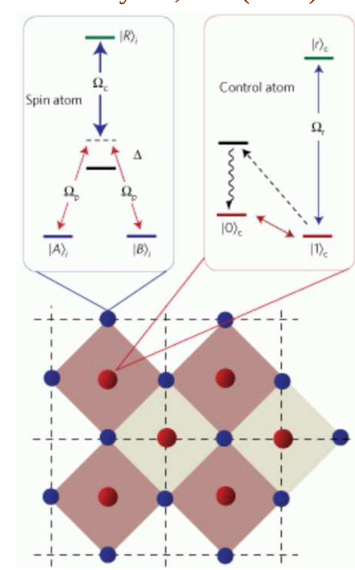
Nat. Phys. 6, 382 (2010)



Universal quantum simulator

Discretize *space* & *time* [ST $e^{\delta(\mathcal{A}+\mathcal{B})} = e^{\delta\mathcal{A}}e^{\delta\mathcal{B}} + O(\delta^2)$]

$\Rightarrow$ spin lattice with finite *range* & *interval* interactions

$$\mathcal{U}(t) = \exp(-\tfrac{i}{\hbar}\mathcal{H}t) \simeq \prod_j \exp(-\tfrac{i}{\hbar}\mathcal{H}_j\delta t_1)\prod_{j'} \exp(-\tfrac{i}{\hbar}\mathcal{H}_{j'}\delta t_2)\ldots$$

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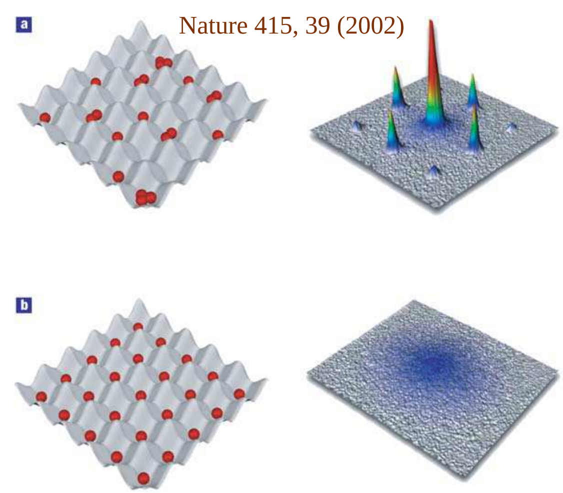
Analog quantum simulator



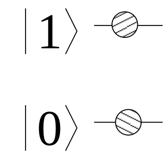
Construct a clean system &
realize appropriate interactions to mimic $\mathcal{H}_S$

$\Rightarrow$ Dynamically or adiabatically evolve $|\Psi\rangle$
and read-out

Quantum computations

Quantum bit – **qubit** – is two-state quantum system

stores superposition states $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$



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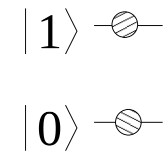

N-qubit memory has 2^N orthogonal states $|x\rangle = |00\ldots0\rangle, \ldots, |11\ldots1\rangle$

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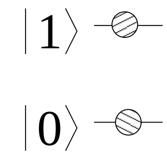

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Any computation (unitary transformation) can be realized via a sequence of one-, two- (and more-) qubit gates

- One-qubit gates (rotations on BS): Unity I, Hadamard H, Pauli X, Y, Z, Phase S ...
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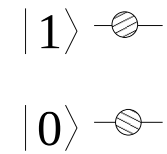

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- Prepare the input state in a superposition state of all possible “classical” inputs x: $|\Psi_{\text{input}}\rangle = \sum_x c_x\,|x\rangle$
- Design an algorithm where all the computational paths interfere with each other to yield with high probability the output state y: $|\Psi_{\text{output}}\rangle \simeq |y\rangle$ $\left(\sum_{y' \neq y} |c_{y'}|^2 \to 0\right)$

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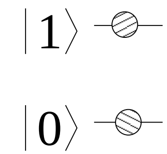

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Useful quantum algorithms:

Grover search of $M = 2^N$ elements in $\sqrt{M}$ steps

Shor FFT for a set $M = 2^N$ in $\sim N^2$ steps (factorization etc.) ...

Janne Rydberg

Rydberg atoms

JANNE RYDBERG

Rydberg atoms

Niels Henrik David Bohr

Rydberg Atoms

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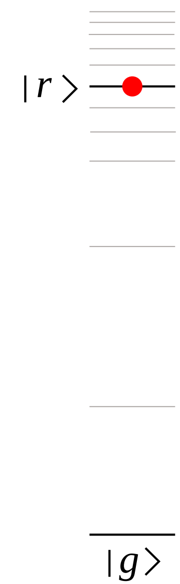

Energy $$\boxed{E_r = -\frac{\mathrm{Ry}}{n^{*2}}}$$

effective PQN $n^* = n - \delta_l$ (δ_l quantum defect)

Rydberg Atoms

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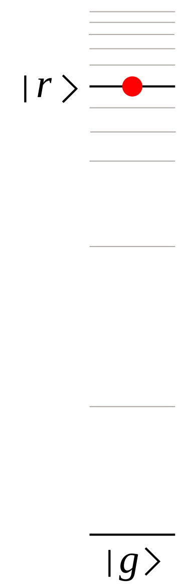

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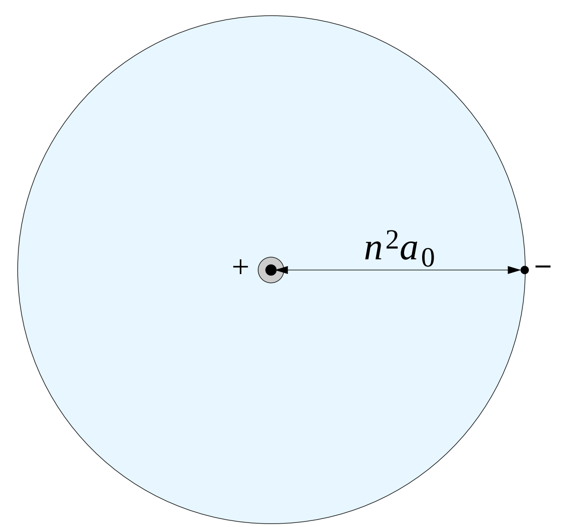
Easily polarized



Huge dipole moments $$\wp \sim n^2 e a_0$$

Gallagher, *Rydberg Atoms* (Cambridge 1994)

Dipole-Dipole Interactions

$$D = \frac{\boldsymbol{\wp}^{(1)} \cdot \boldsymbol{\wp}^{(2)}}{R^3} - 3\frac{(\boldsymbol{\wp}^{(1)} \cdot \boldsymbol{R})(\boldsymbol{\wp}^{(2)} \cdot \boldsymbol{R})}{R^5} \propto n^4$$

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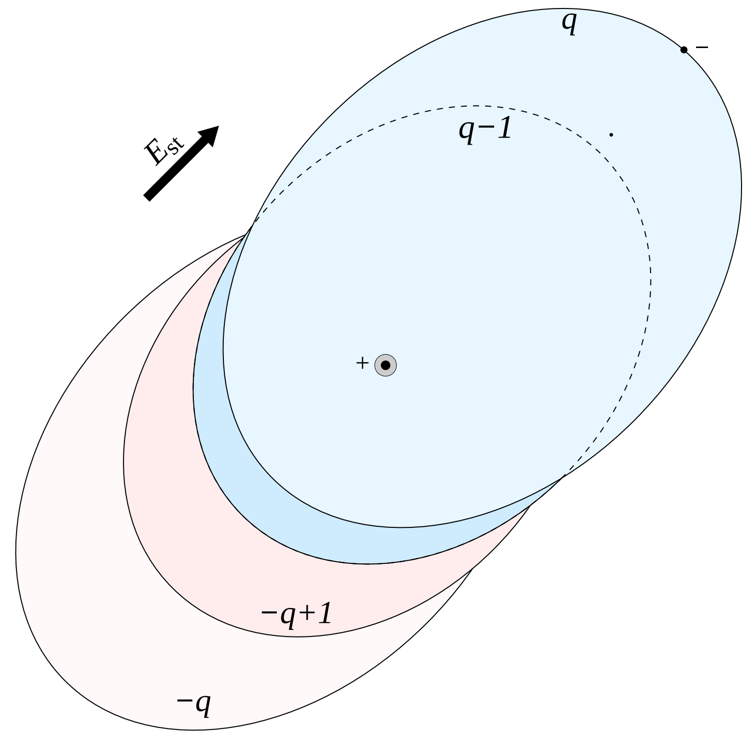
⇒ **Static DDI**

$\boldsymbol{E}_{\mathrm{st}}$ induced Stark eigenstates

with permanent $\wp = \frac{3}{2} nqea_0$



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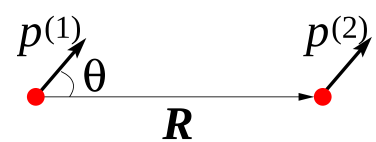

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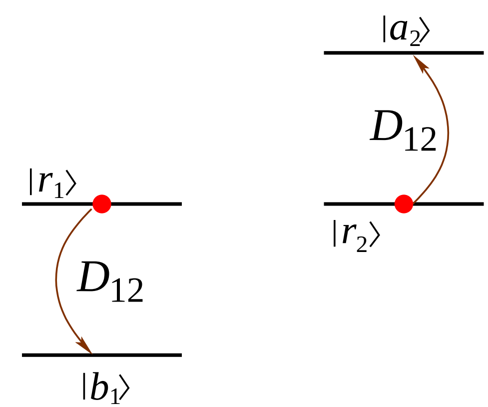
⇒ **Resonant DDI**

$$D = \frac{1}{R^3}\Big[\wp_{+1}^{(1)}\wp_{-1}^{(2)} + \wp_{-1}^{(1)}\wp_{+1}^{(2)} + \wp_{0}^{(1)}\wp_{0}^{(2)}(1 - 3\cos^2\theta)$$
$$-\frac{3}{2}\sin^2\theta(\wp_{+1}^{(1)}\wp_{+1}^{(2)} + \wp_{+1}^{(1)}\wp_{-1}^{(2)} + \wp_{-1}^{(1)}\wp_{+1}^{(2)} + \wp_{-1}^{(1)}\wp_{-1}^{(2)}$$
$$-\frac{3}{\sqrt{2}}\sin\theta\cos\theta(\wp_{+1}^{(1)}\wp_{0}^{(2)} + \wp_{-1}^{(1)}\wp_{0}^{(2)} + \wp_{0}^{(1)}\wp_{+1}^{(2)} + \wp_{0}^{(1)}\wp_{-1}^{(2)})\Big]$$



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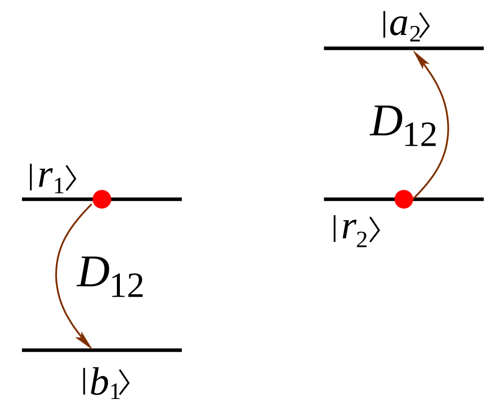

$$D = \frac{1}{R^3}\Big[\wp^{(1)}_{+1}\wp^{(2)}_{-1} + \wp^{(1)}_{-1}\wp^{(2)}_{+1} + \wp^{(1)}_{0}\wp^{(2)}_{0}(1 - 3\cos^2\theta)$$
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$$-\frac{3}{\sqrt{2}}\sin\theta\cos\theta(\wp^{(1)}_{+1}\wp^{(2)}_{0} + \wp^{(1)}_{-1}\wp^{(2)}_{0} + \wp^{(1)}_{0}\wp^{(2)}_{+1} + \wp^{(1)}_{0}\wp^{(2)}_{-1})\Big]$$

$$\wp_{+1} = -\frac{e}{\sqrt{2}}(\hat{x} + i\hat{y}) = -\frac{e}{\sqrt{2}}r\sin\theta e^{i\phi} = er\sqrt{\frac{4\pi}{3}}Y_{1,1}(\theta,\phi) \qquad r_{+1} = \langle n'l', m+1|\, r\, |nl, m\rangle$$

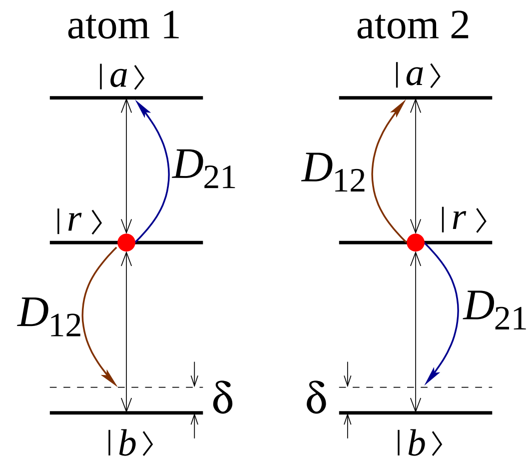
$$\wp_{0} = e\hat{z} = er\cos\theta = er\sqrt{\frac{4\pi}{3}}Y_{1,0}(\theta,\phi) \qquad r_{0} = \langle n'l', m|\, r\, |nl, m\rangle$$

$$\wp_{-1} = \frac{e}{\sqrt{2}}(\hat{x} - i\hat{y}) = \frac{e}{\sqrt{2}}r\sin\theta e^{-i\phi} = er\sqrt{\frac{4\pi}{3}}Y_{1,-1}(\theta,\phi) \qquad r_{-1} = \langle n'l', m-1|\, r\, |nl, m\rangle$$

van der Waals Interaction

RDDI (Förster process)

$$D_{12} \equiv D(R) \propto \frac{\wp_{br}\wp_{ar}}{R^3} \propto n^4$$



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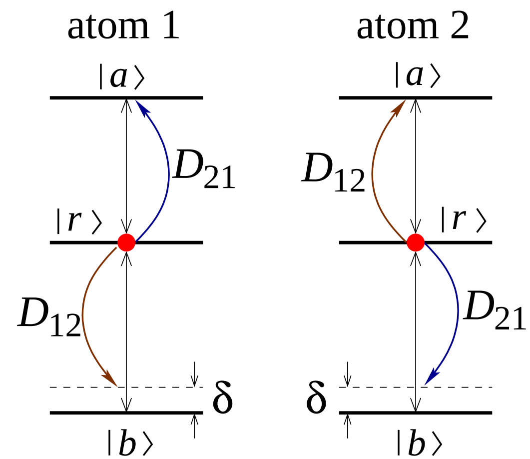
$(\delta\omega \propto n^{-3})$



$\Rightarrow$ $|r_1\rangle\,|r_2\rangle \nrightarrow |a_{1,2}\rangle\,|b_{2,1}\rangle$: **Non-Resonant DDI** (Adiabatic elim. $|a_{1,2}\rangle\,|b_{2,1}\rangle$)

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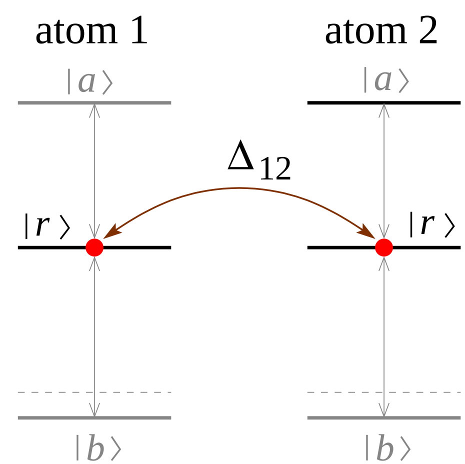

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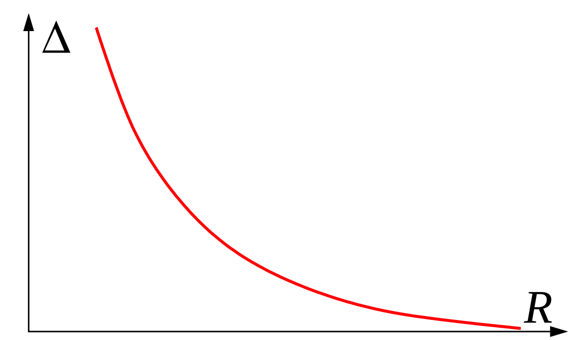
$\Rightarrow$ Energy shift of $|r_1\rangle\,|r_2\rangle$ (2nd-order in $D/\delta\omega$)

$$\mathcal{V}_{\mathrm{vdW}} = \hbar\hat{\sigma}^1_{rr}\Delta_{12}\hat{\sigma}^2_{rr}$$

$$\Delta_{12} \equiv \Delta(R) = 2\frac{|D(R)|^2}{\delta\omega} = \frac{C_6}{R^6} \propto n^{11}$$ — vdWI strength



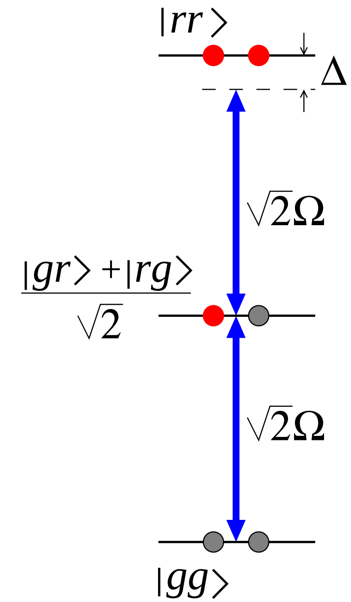
Saffman, Walker, Mølmer, RMP **82**, 2313 (2010)

Two Atoms

Rydberg interaction blockade

Two-atom Hamiltonian

$$\mathcal{H}/\hbar = \Omega\,|r\rangle_1\langle g|\, + \Omega\,|r\rangle_2\langle g|\, + \text{H.c.}$$
$$+\Delta_{12}\,|r\rangle_1\langle r|\,\otimes\,|r\rangle_2\langle r|$$



Rydberg interaction blockade

Two-atom Hamiltonian

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Resonant transition $|gg\rangle \leftrightarrow |gr, rg\rangle$

$$\frac{1}{\hbar}\langle gr, rg| \, \mathcal{H} \, |gg\rangle = \Omega$$

Nonresonant transition $|gr, rg\rangle \leftrightarrow |rr\rangle$

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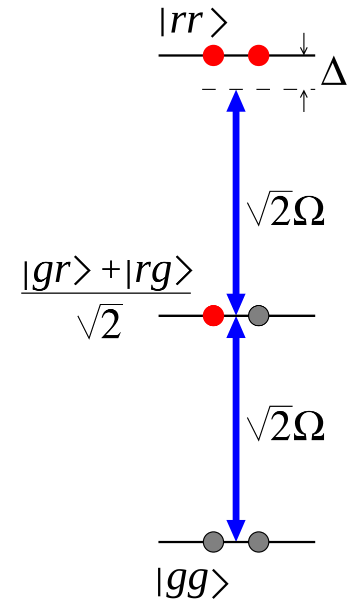

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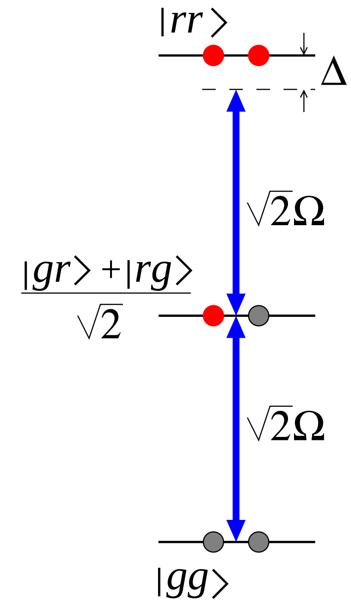

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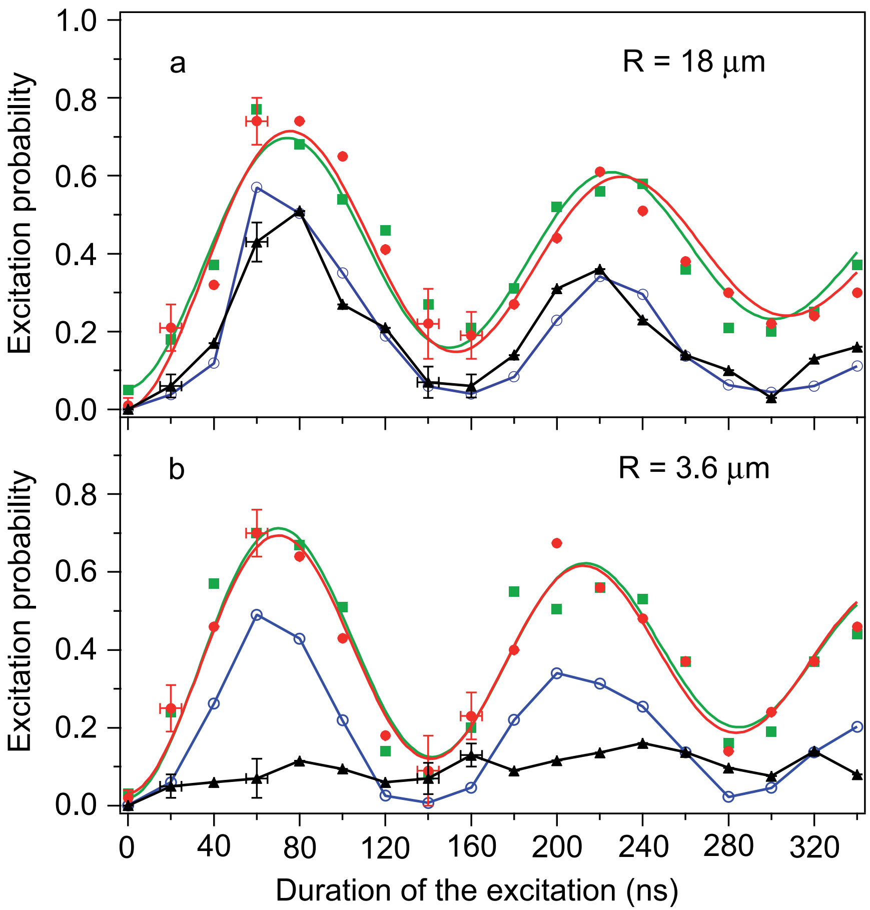
$\Rightarrow$ **Double excitation** $|rr\rangle$ **is blocked**

$\Rightarrow$ **Rabi oscillations** $|gg\rangle \leftrightarrow \frac{1}{\sqrt{2}}(\,|gr\rangle + |rg\rangle)$ with $\sqrt{2}\Omega$

Jaksch *et al.*, PRL **85**, 2208 (2000)

Two-atom Rabi oscillations



Urban *et al.*, Nature Phys. **5**, 110 (2009); Gaëtan *et al.*, Nature Phys. **5**, 115 (2009)

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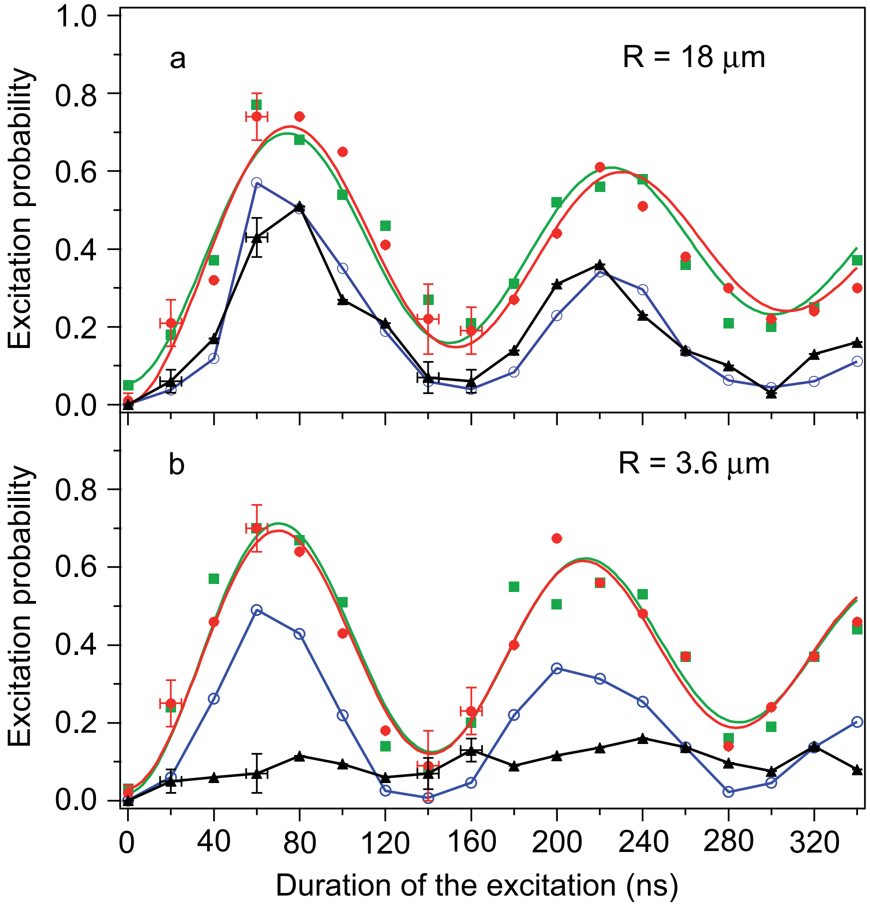

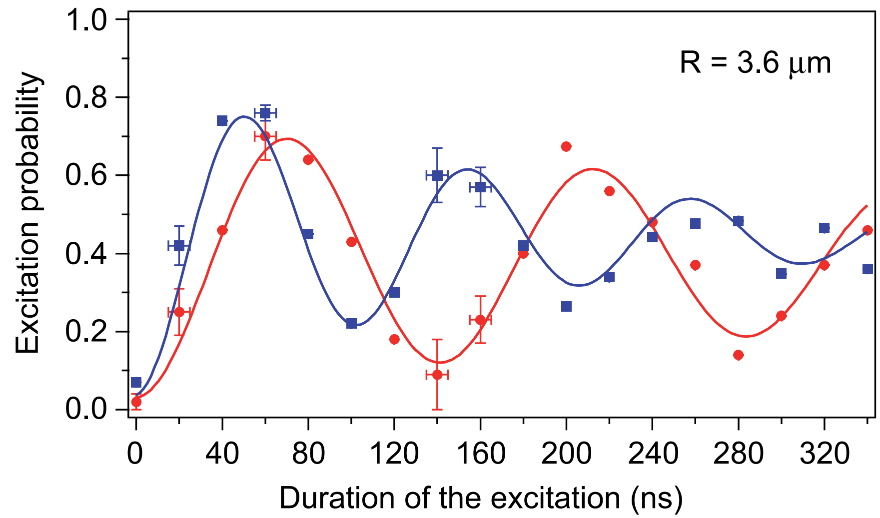

Urban *et al.*, Nature Phys. **5**, 110 (2009); Gaëtan *et al.*, Nature Phys. **5**, 115 (2009)

Rydberg blockade gates

Two-atom (qubit) CZ gate

$$|0\rangle_1\,|0\rangle_2 \rightarrow \quad |0\rangle_1\,|0\rangle_2$$
$$|0\rangle_1\,|1\rangle_2 \rightarrow -\,|0\rangle_1\,|1\rangle_2$$
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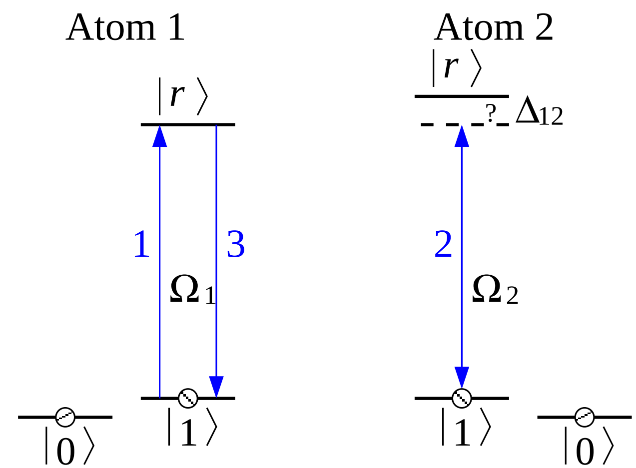

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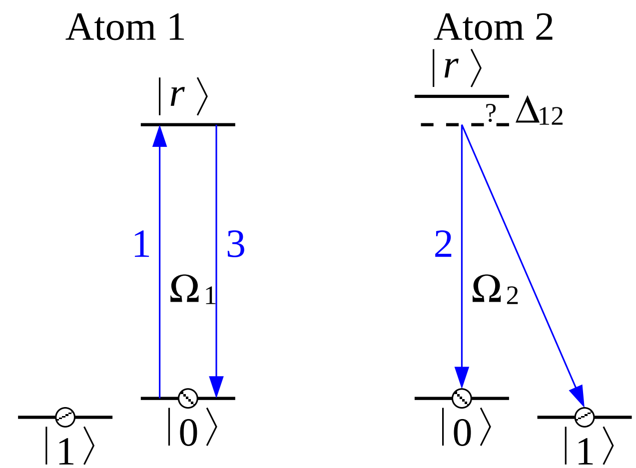

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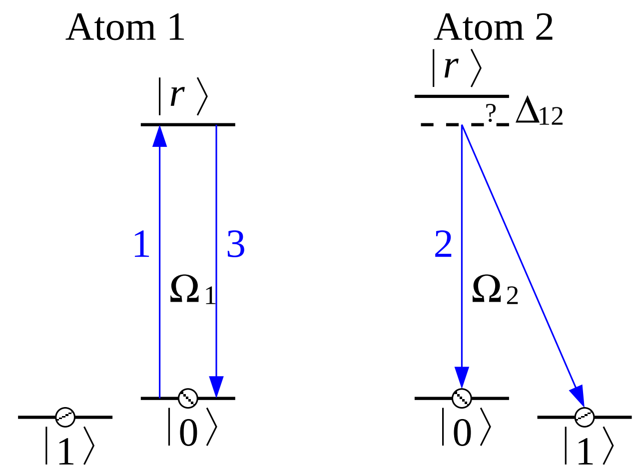
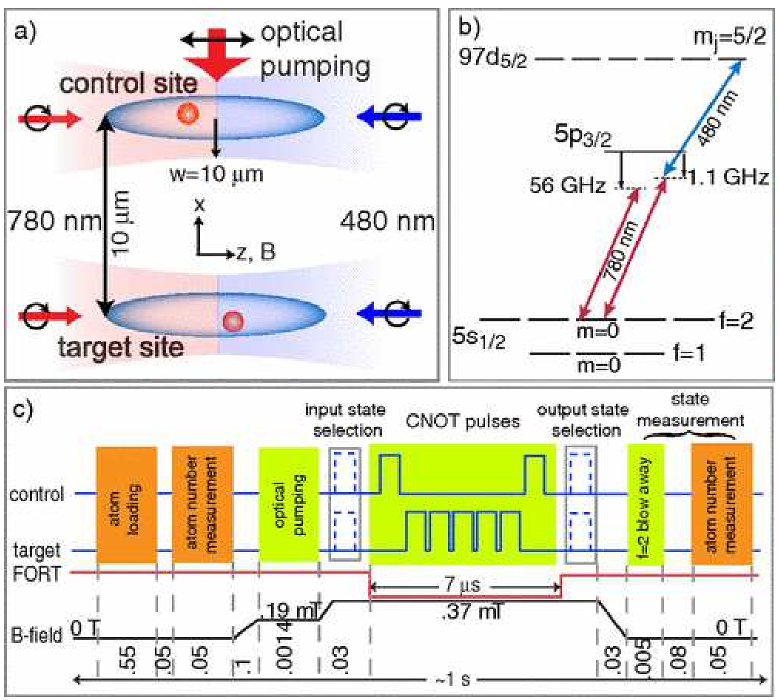
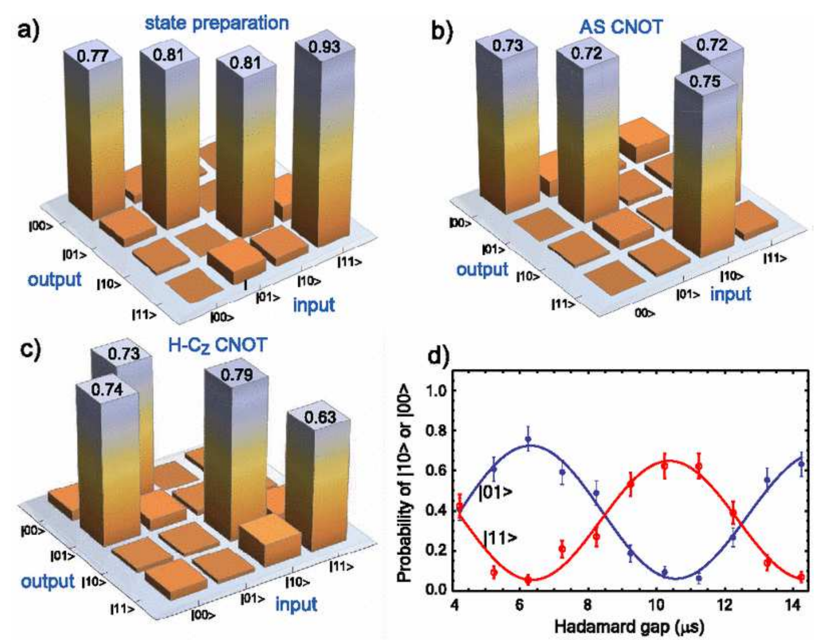


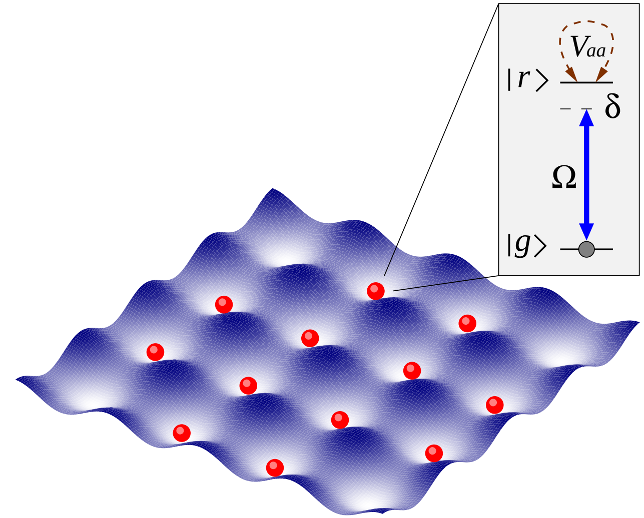




Wilk *et al.*, PRL **104**, 010502 (2010); Isenhower *et al.*, PRL **104**, 010503 (2010)

Spatially extended systems

Lattice spin models



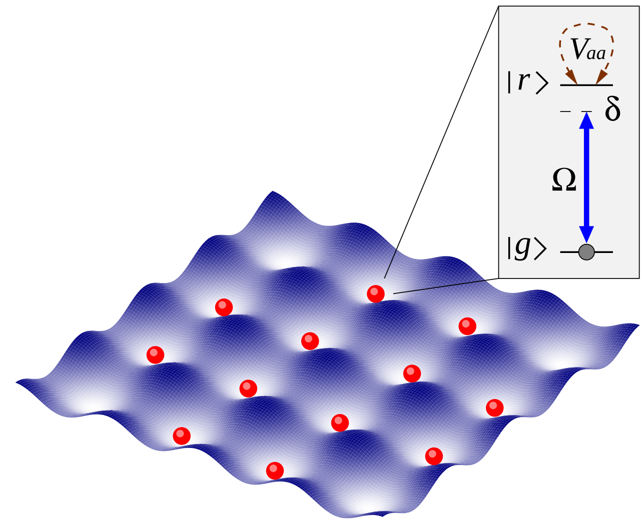
Lattice spin models

spin-$\frac{1}{2}$ Hamiltonian (Ising-like)

$$\mathcal{H}_{\rm spin} = \hbar \sum_j^N [\Omega \hat{\sigma}_x^j - \frac{\delta_j}{2} \hat{\sigma}_z^j] + \frac{\hbar}{4} \sum_{i \neq j}^N \Delta_{ij} \hat{\sigma}_z^i \hat{\sigma}_z^j$$

with Ω, $\delta_j = (\delta - \frac{1}{2} \sum_{i \neq j} \Delta_{ij})$ & $\Delta_{ij} \equiv \Delta(\mathbf{x}_i - \mathbf{x}_j)$

- isotropic or anisotropic interaction
- DD ($\propto 1/R^3$) or vdW ($\propto 1/R^6$)



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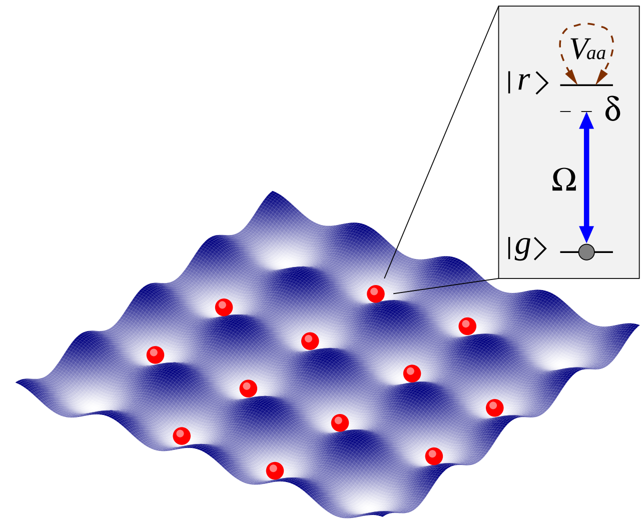
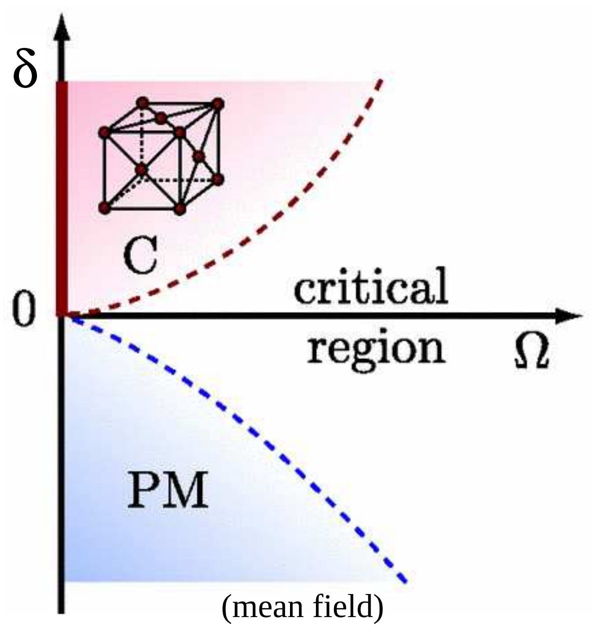


Phase diagram



Weimer et al., PRL **101**, 250601 (2008)

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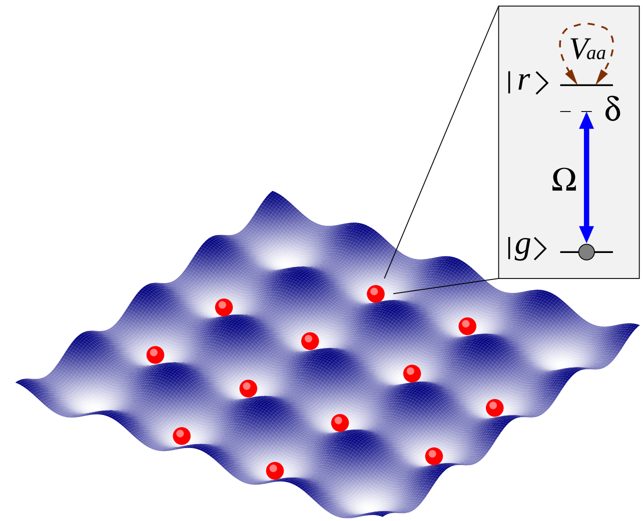

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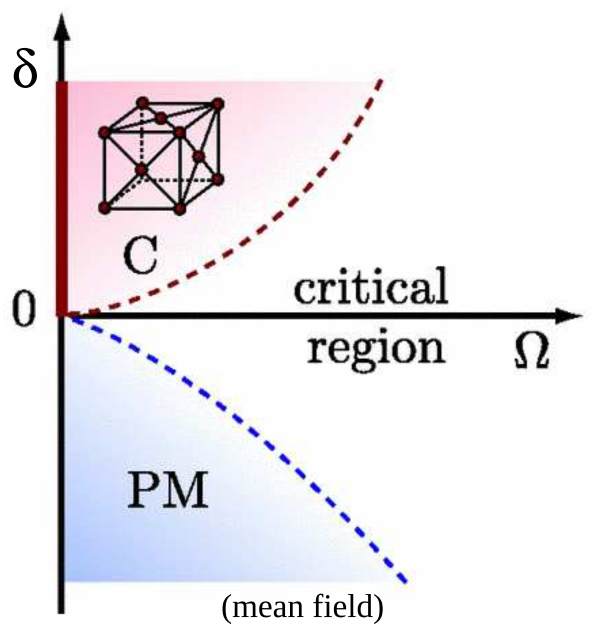
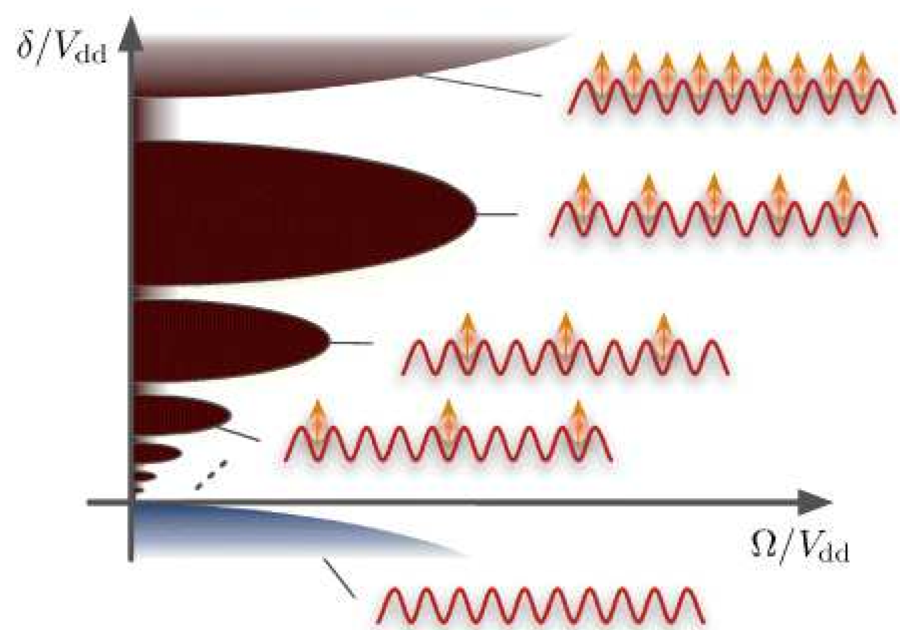
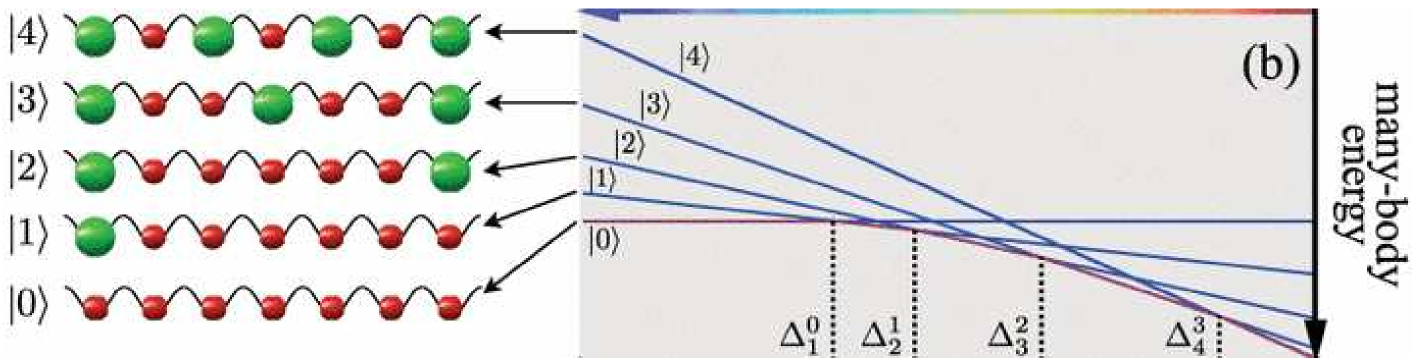


Weimer et al., PRL **101**, 250601 (2008)



Schachenmayer et al, NJP **12**, 103044 (2010)

Dynamical crystal preparation (1D)



$$\Omega \to 0$$

$$\mathcal{H}/\hbar = -\delta \sum_j^N \hat{\sigma}_{rr}^j - \Omega \sum_j^N (\hat{\sigma}_{rg}^j + \hat{\sigma}_{gr}^j) + \sum_{i<j}^N \Delta_{ij} \hat{\sigma}_{rr}^i \hat{\sigma}_{rr}^j$$

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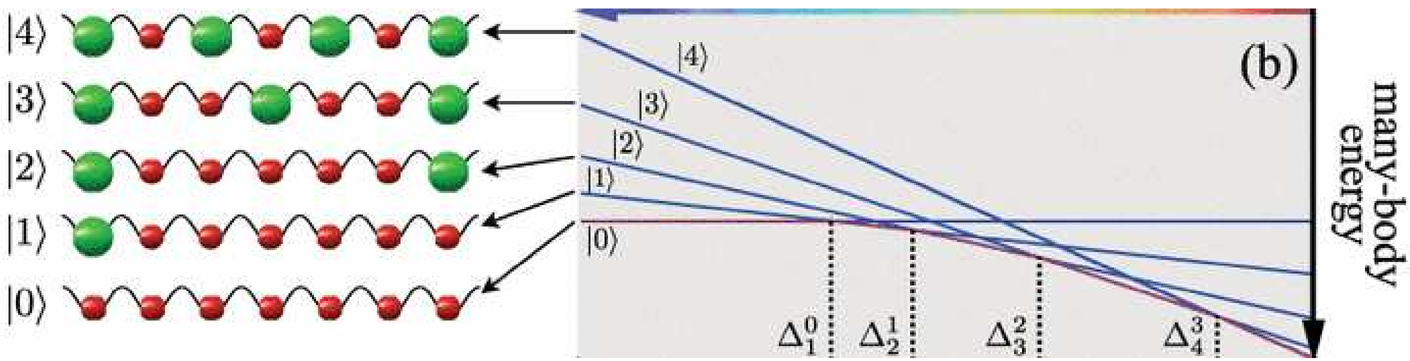

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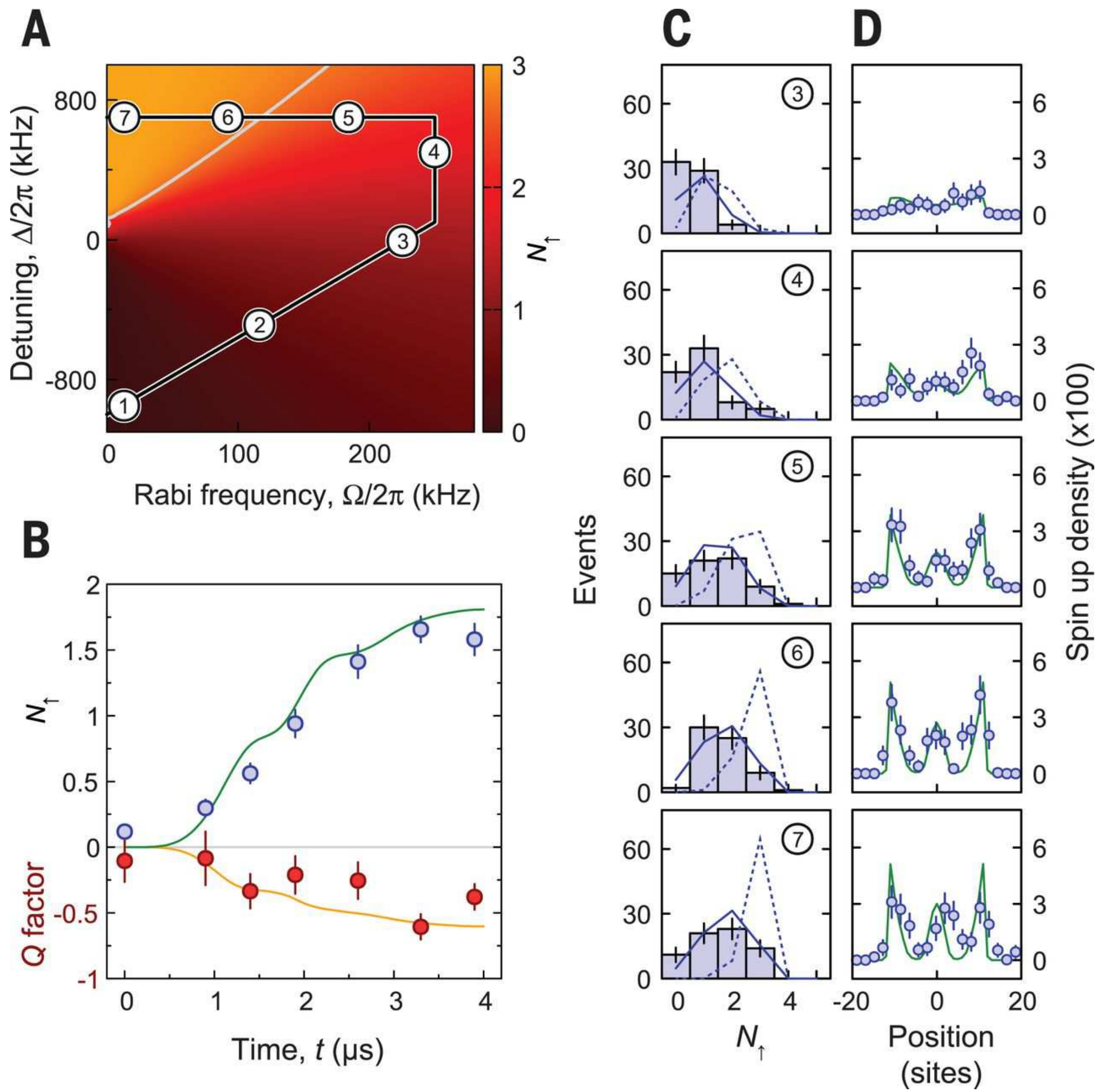
Avoided crossings $\pm\Omega_{n+1}^n \neq 0$:

$$\Omega_1^0 = \sqrt{N}\Omega, \;\; \Omega_2^1 = 2\Omega/\sqrt{N}, \;\; \Omega_3^2 = \Omega, \;\; \Omega_4^3 \simeq \Omega^3/[C_6/(l/3)^6]^2 \ldots$$

Pohl, Demler, Lukin, PRL **104**, 043002 (2010)
Schachenmayer, Lesanovsky, Micheli, Daley, NJP **12**, 103044 (2010)
Petrosyan, Mølmer, Fleischhauer, JPB **49**, 084003 (2016)

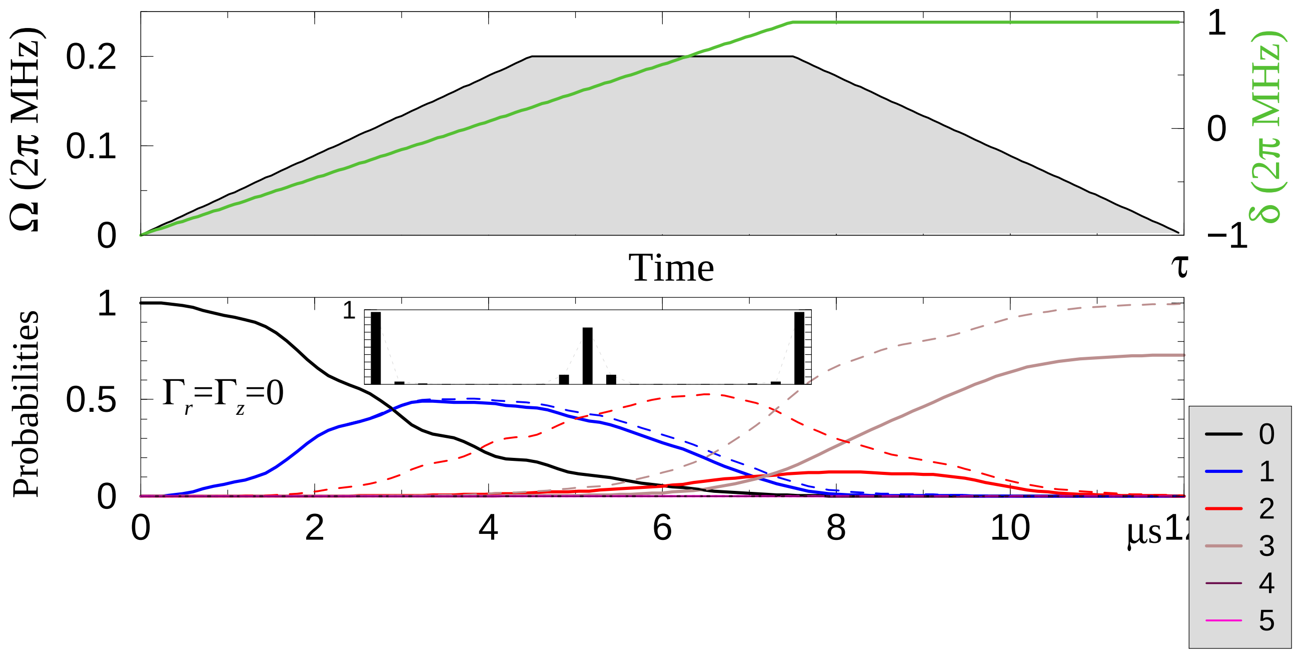
Dynamical crystal preparation (1D)

Experiment



Schauß et al., Science **347** 1455 (2015); Bernien et al., arXiv:1707.04344

Simulations of adiabatic dynamics (QMC WF)



$$P_n^{\min} \equiv \langle R_n^{\min} | \hat{\rho} | R_n^{\min} \rangle$$

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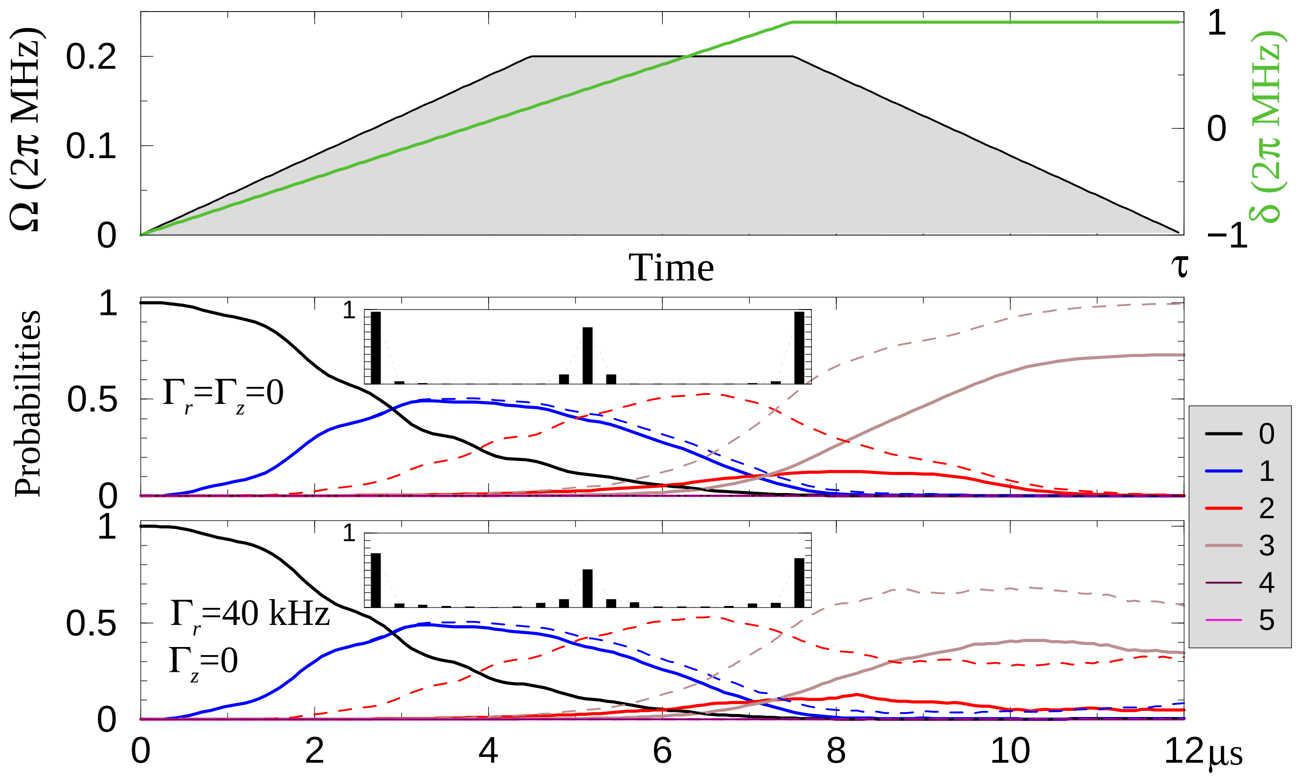

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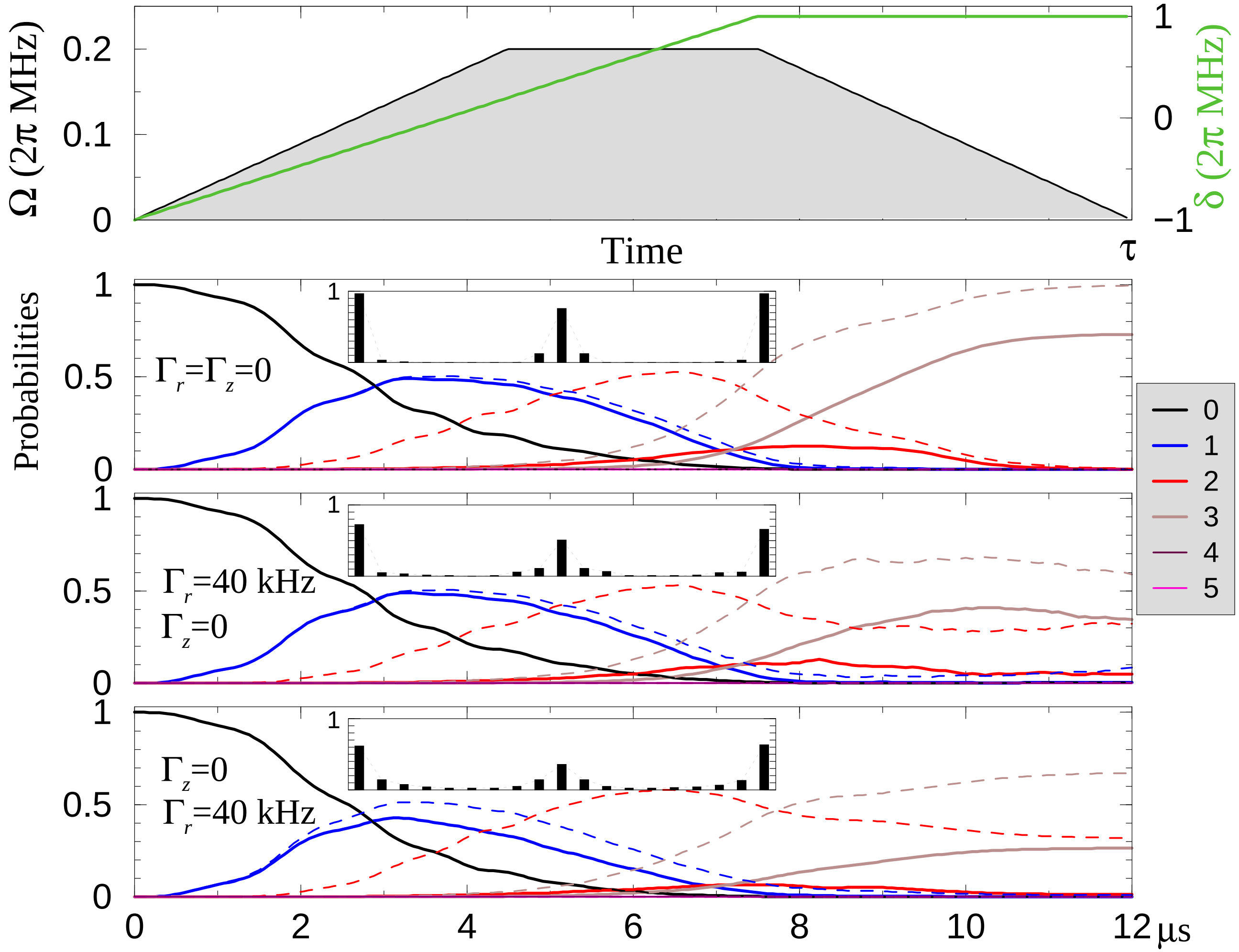

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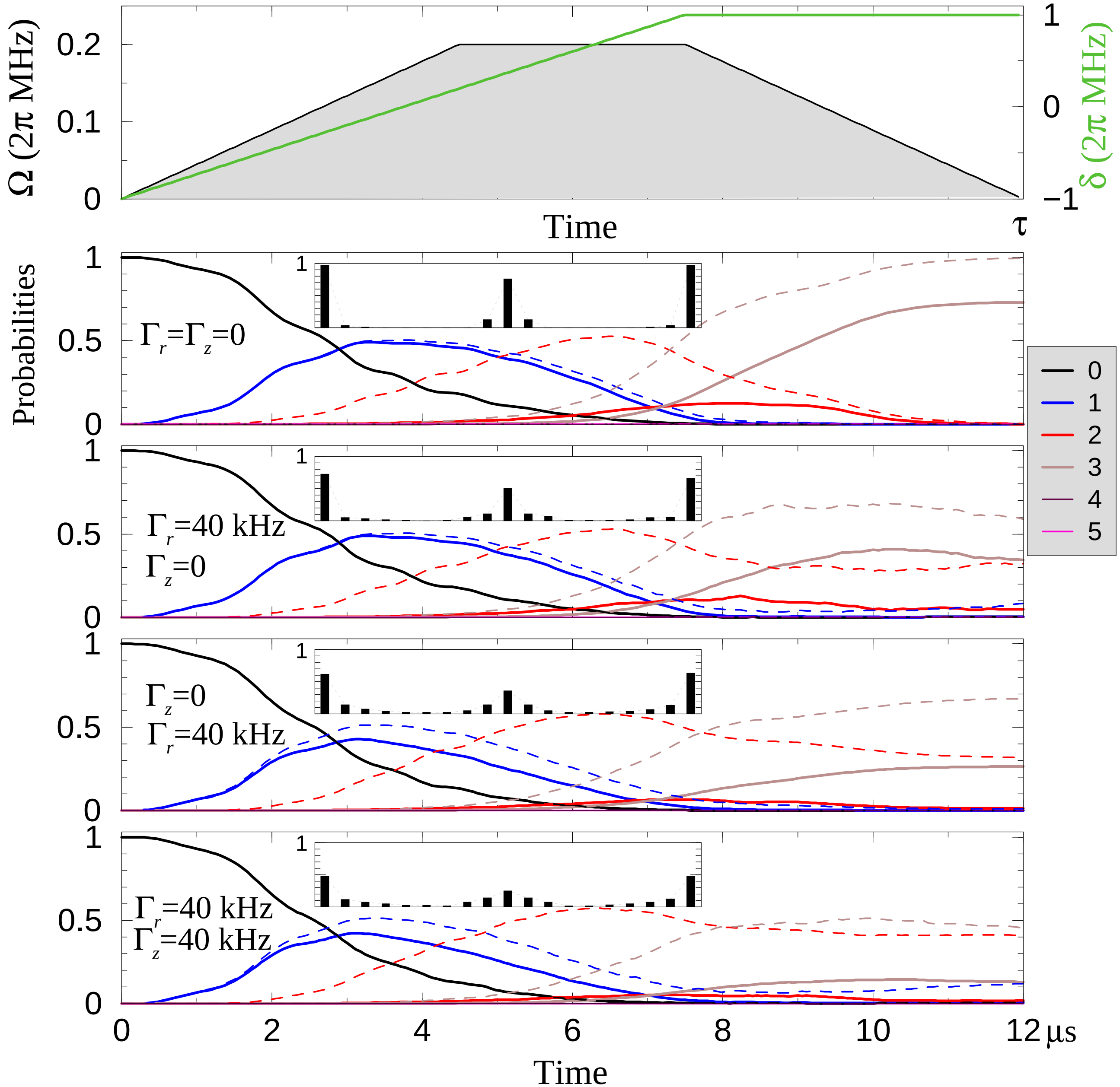

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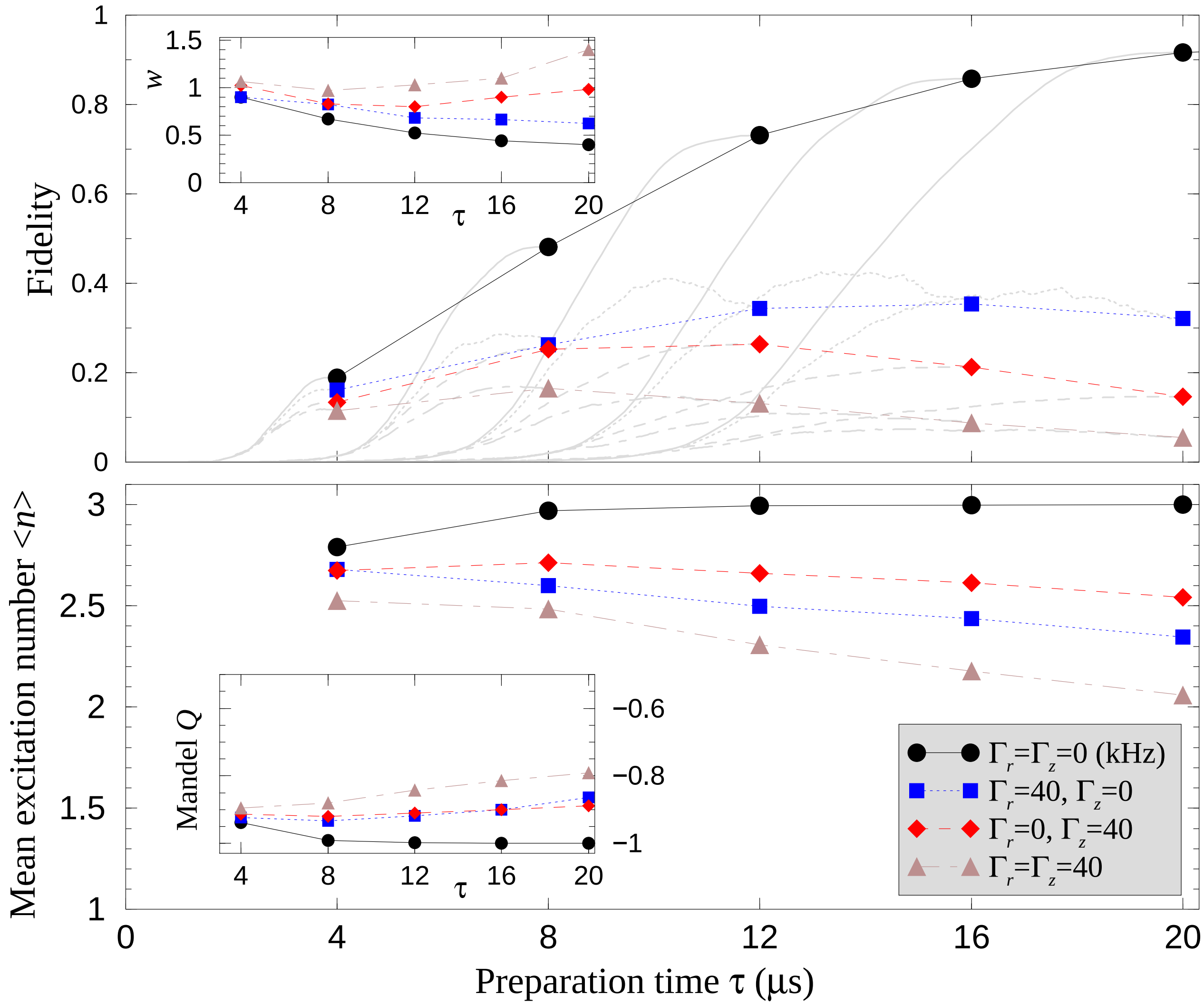

$$F = P_3^{\min}$$

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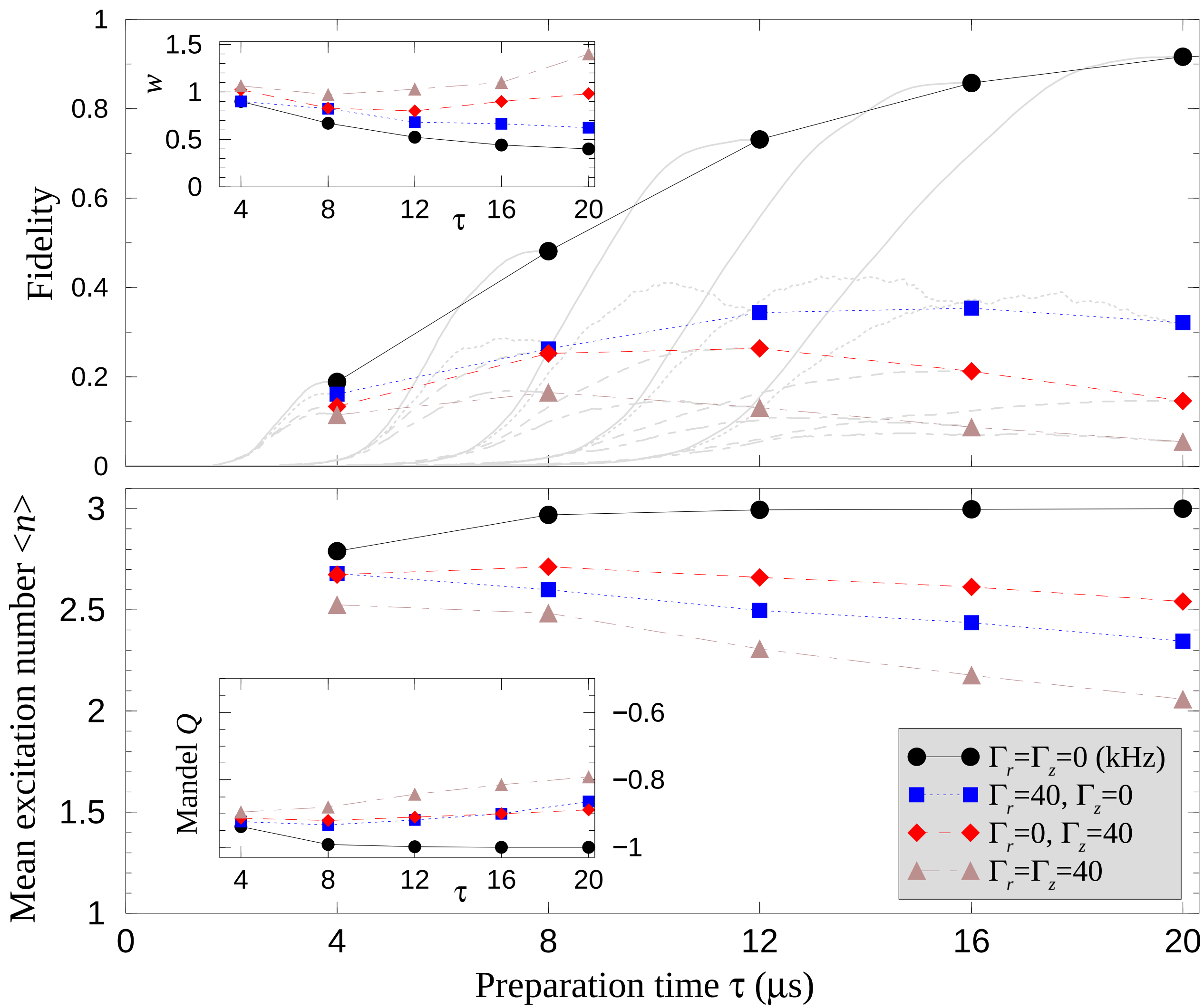

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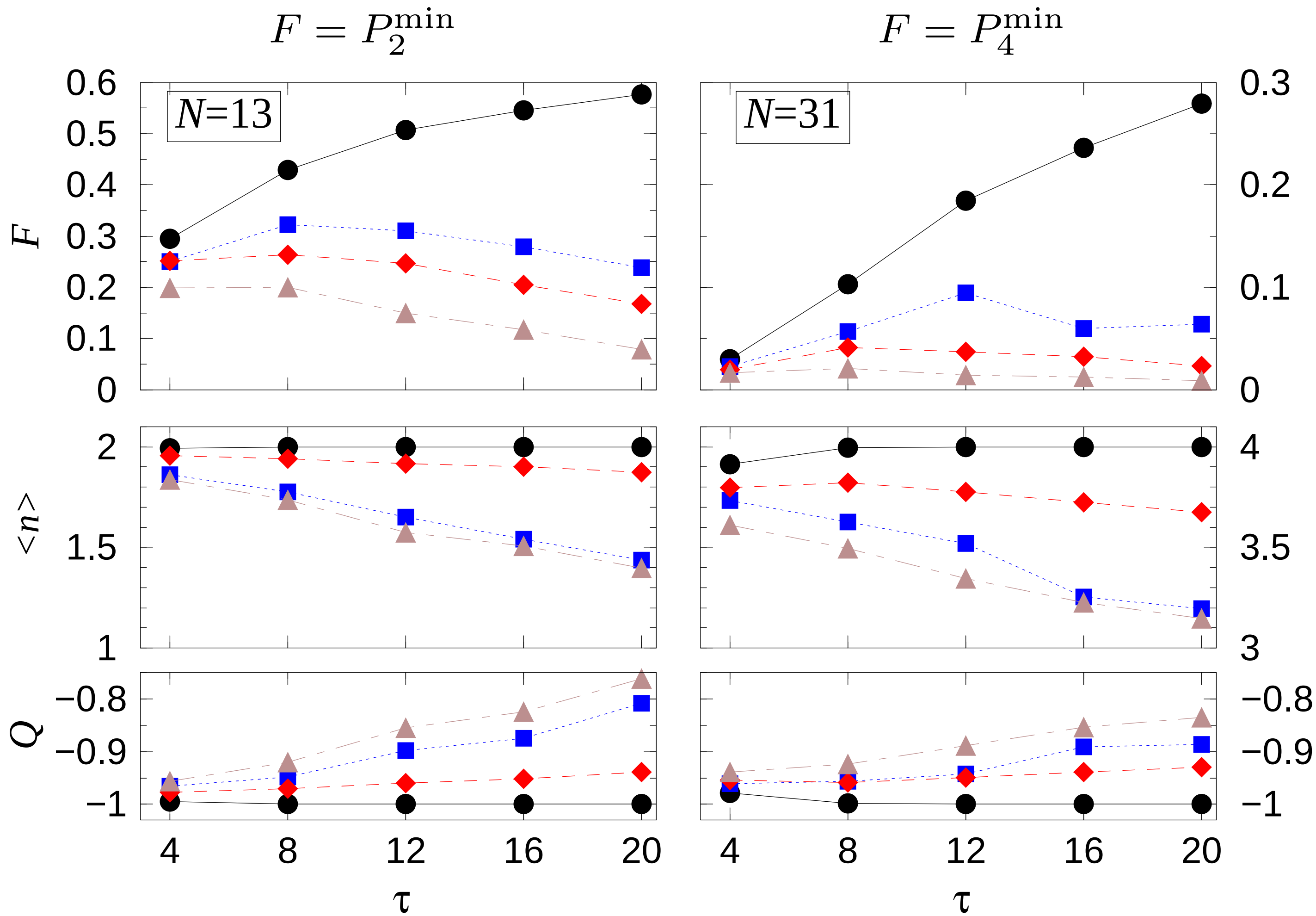
Relaxations destroy adiabatic following of the ground state

$$|R_0\rangle \to |R_1\rangle \to |R_2^{\min}\rangle \to |R_3^{\min}\rangle$$

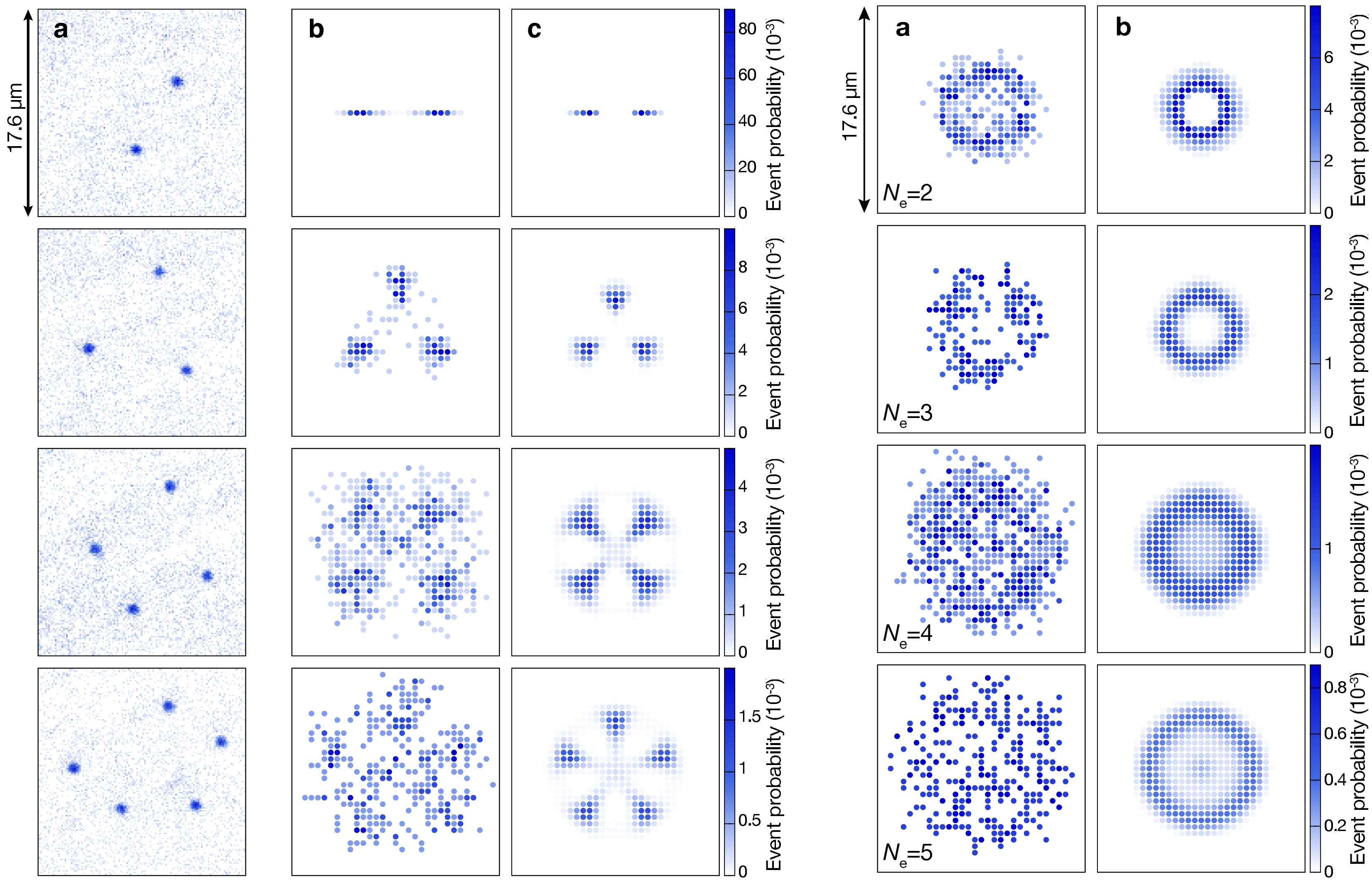
Simulations of adiabatic dynamics



Petrosyan, Mølmer, Fleischhauer, JPB **49**, 084003 (2016)

Extended 2D systems

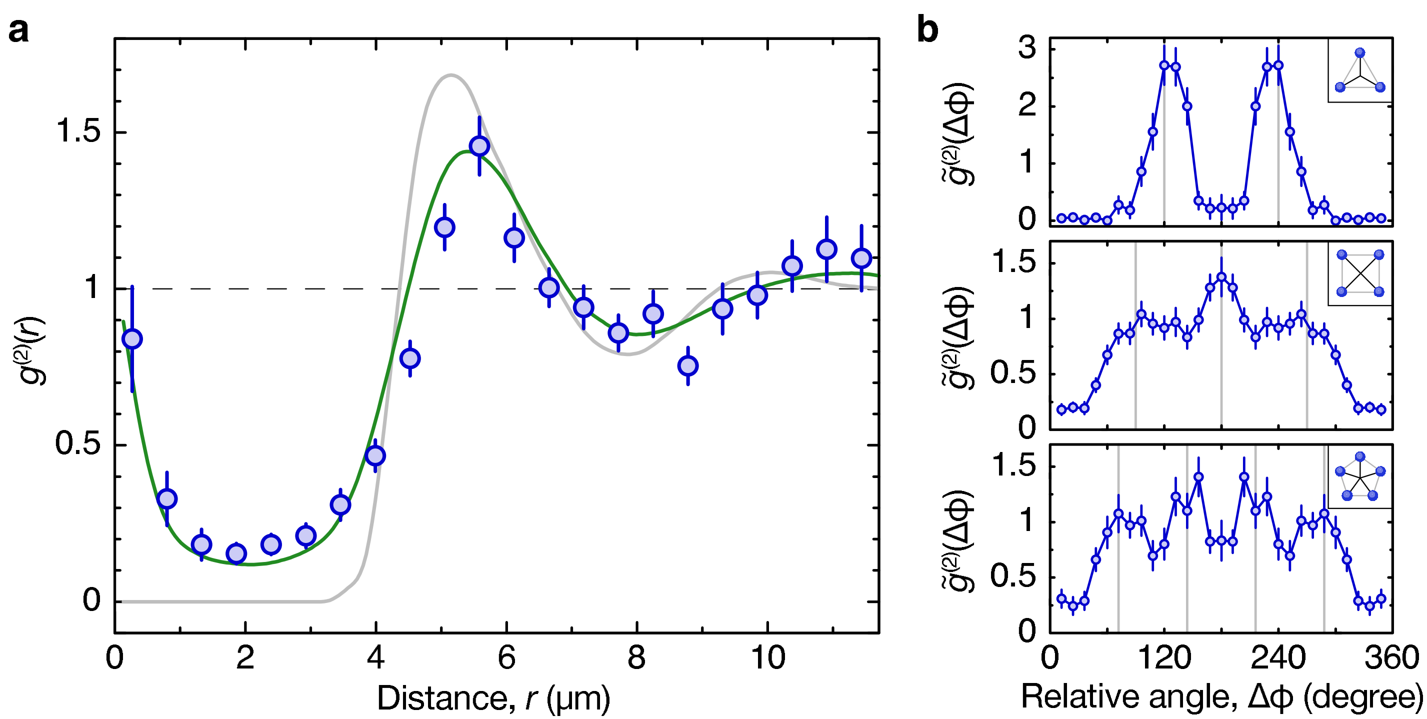
Rydberg Quasi-Crystals (2D)



$$a_{\rm lat} = 0.532\ \mu\text{m}, \quad d_{\rm b} = 3.81\ \mu\text{m}$$

Schauß *et al.*, Nature **491**, 87 (2012)

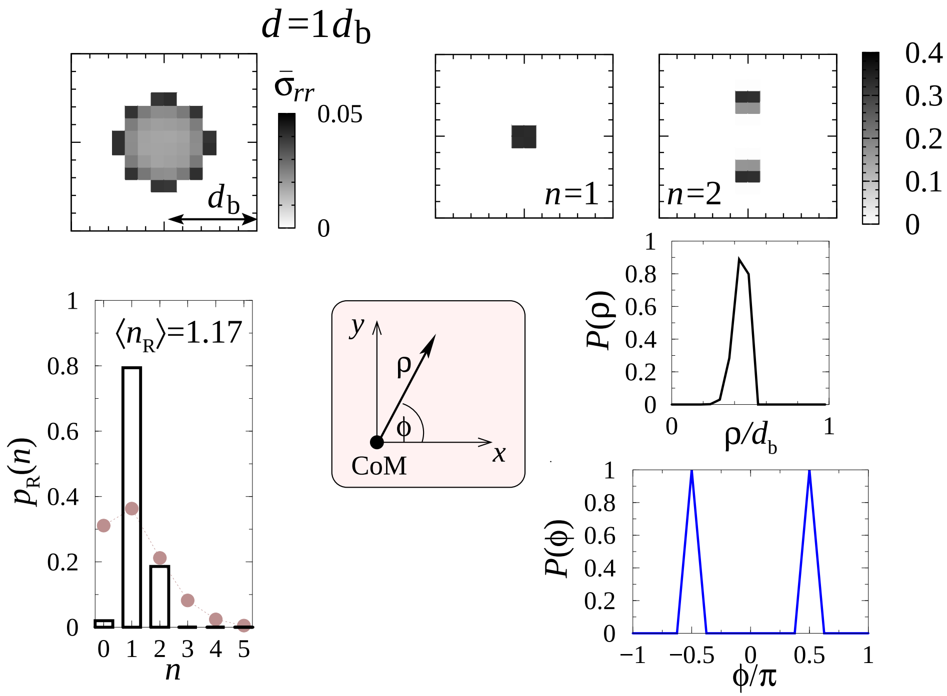
Spatial and Angular Correlations



$$g^{(2)}(r) = \frac{\sum_{i\neq j} \delta_{r,r_{ij}} \langle \hat{\sigma}^i_{rr} \hat{\sigma}^j_{rr} \rangle}{\sum_{i\neq j} \delta_{r,r_{ij}} \langle \hat{\sigma}^i_{rr} \rangle \langle \hat{\sigma}^j_{rr} \rangle} \qquad g^{(2)}(\Delta\phi) = \int \frac{d\phi}{2\pi} \frac{\sum_{i,j} \delta_{\phi,\phi_i} \delta_{\phi+\Delta\phi,\phi_j} \langle \hat{\sigma}^i_{rr} \hat{\sigma}^j_{rr} \rangle}{\sum_{i,j} \delta_{\phi,\phi_i} \langle \hat{\sigma}^i_{rr} \rangle \delta_{\phi+\Delta\phi,\phi_j} \langle \hat{\sigma}^j_{rr} \rangle}$$

Schauß *et al.*, Nature **491**, 87 (2012)

Rydberg Quasi-Crystals



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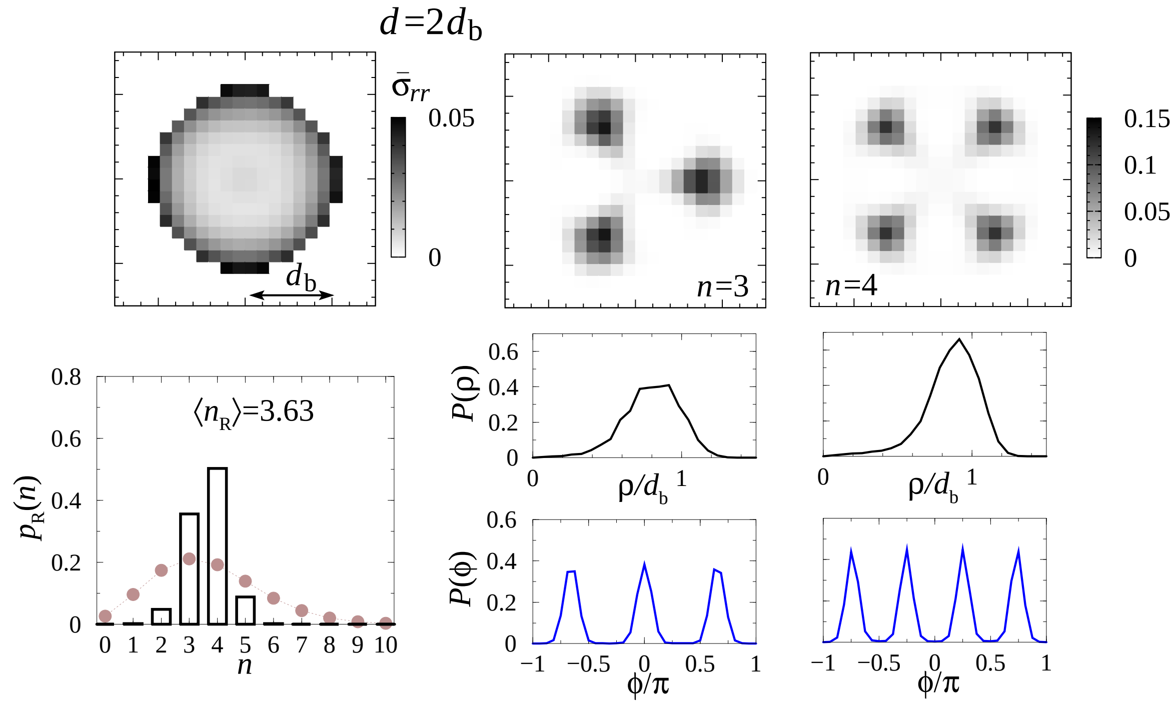

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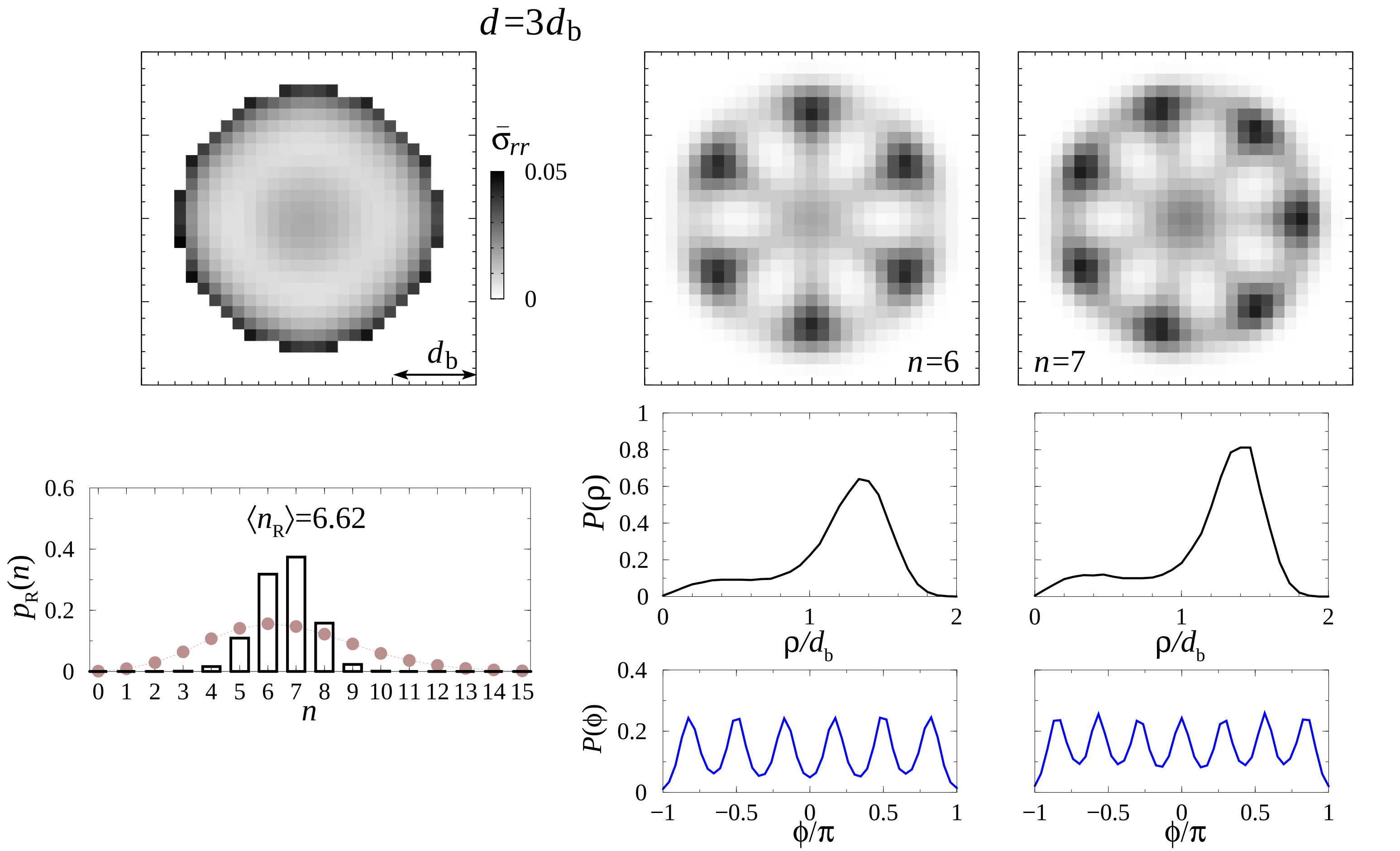

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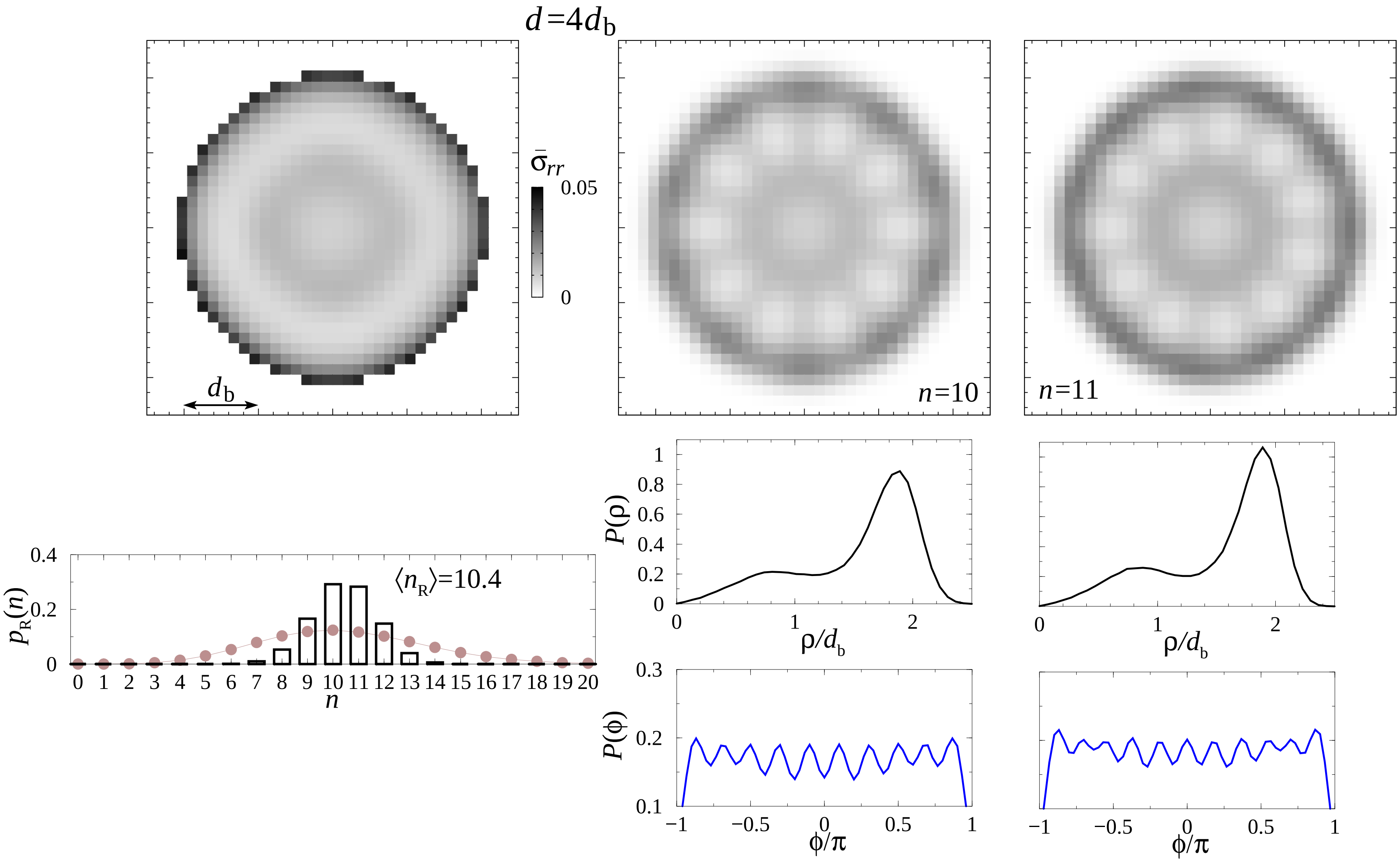

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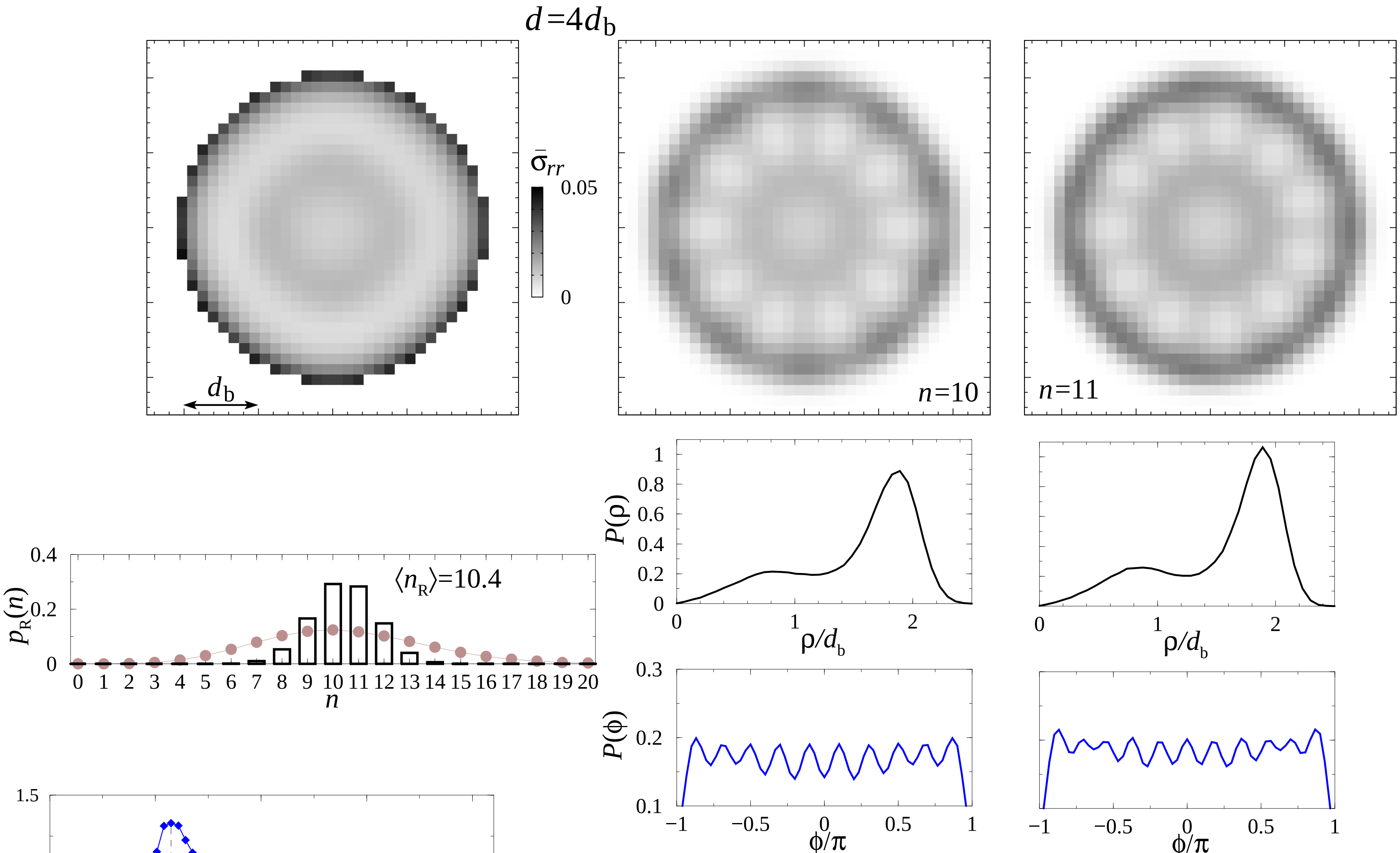

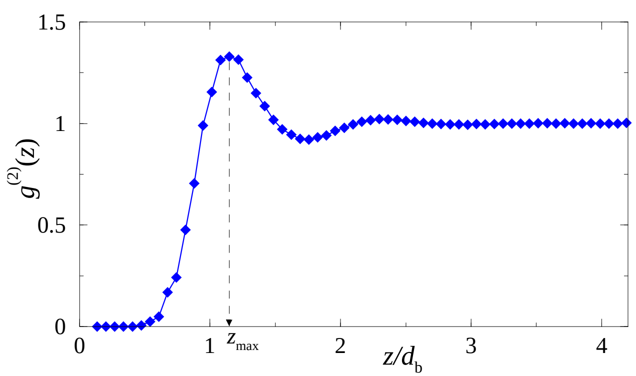

nearly incompressible Liquid of Rydberg exitations

Petrosyan, PRA **88**, 043431 (2013)

Summary: Rydberg states offer

- Strong, long-range, switchable interactions between atoms

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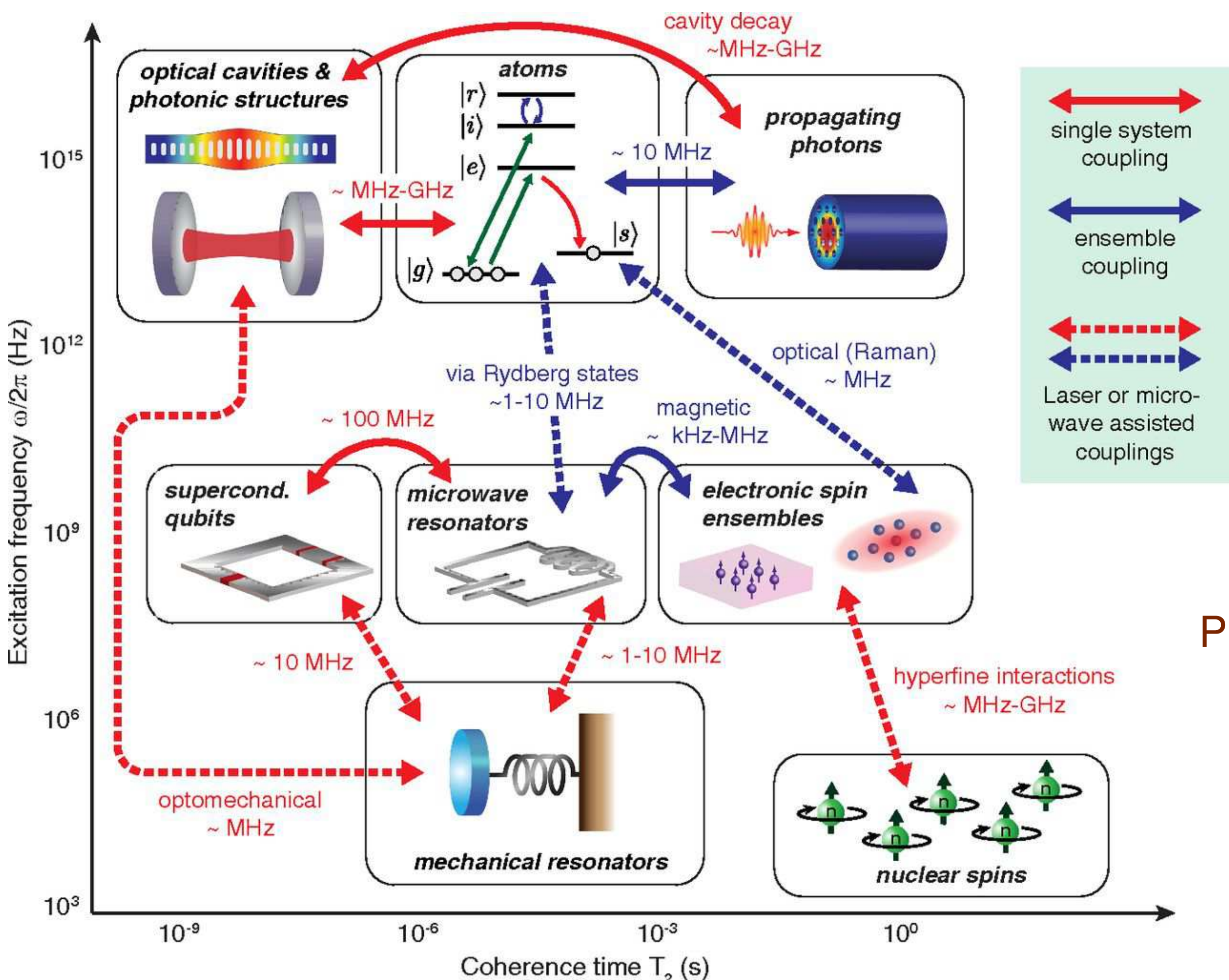

PNAS **112**, 3866 (2015)

Thank you!