



Quantum nonlinear optics mediated by long-range interactions between Rydberg atoms

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Outline



- Long-range Dipole-Dipole (DD) & van der Waals (vdW) interactions between Rydberg atoms

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- Conclusions



Interactions between Rydberg atoms

Rydberg Atoms



High principal quantum number

$$\boxed{n \gg 1} \quad (\text{H-like})$$

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Energy $\boxed{E_r = -\frac{Ry}{n^{*2}}}$

effective PQN $n^* = n - \delta_l$ (δ_l quantum defect)



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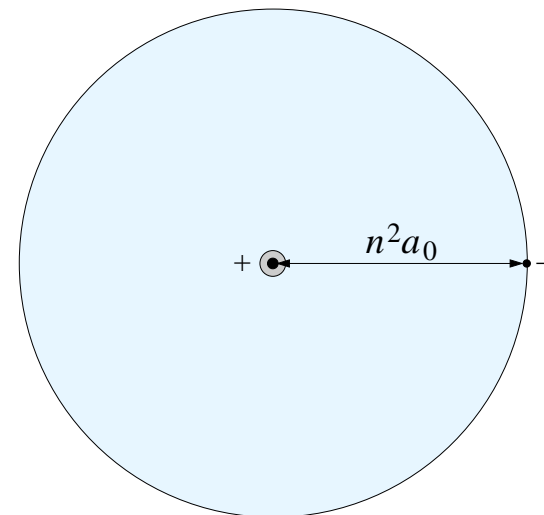
Energy $\boxed{E_r = -\frac{Ry}{n^{*2}}}$

effective PQN $n^* = n - \delta_l$ (δ_l quantum defect)



Easily polarizable

Huge dipole moments $\boxed{\varphi \sim n^2 ea_0}$



Dipole-Dipole Interactions



$$D = \frac{\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2}{R^3} - 3 \frac{(\boldsymbol{\rho}_1 \cdot \mathbf{R})(\boldsymbol{\rho}_2 \cdot \mathbf{R})}{R^5} \propto n^4$$

Dipole-Dipole Interactions

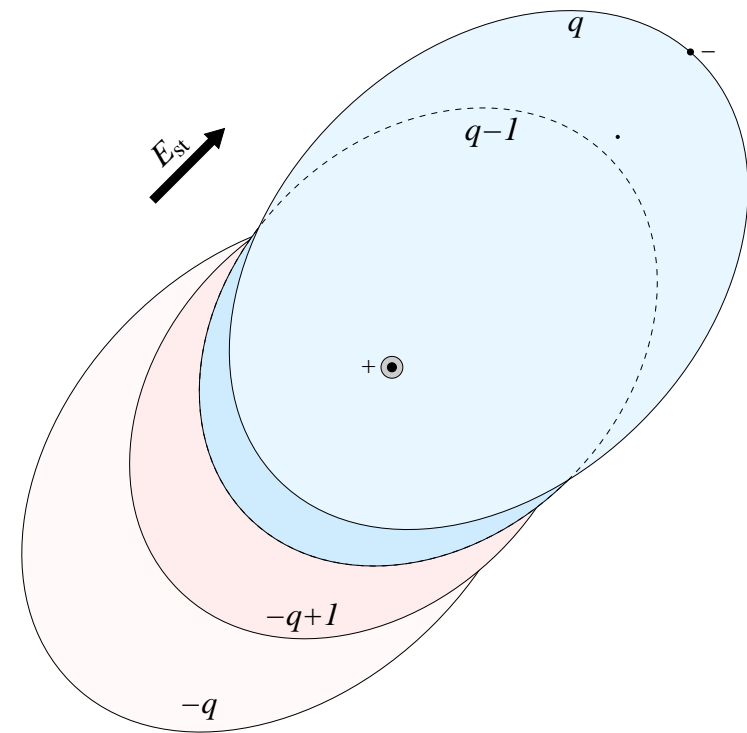


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⇒ **Static DDI**

E_{st} induced Stark eigenstates

with permanent $\boldsymbol{\rho} = \frac{3}{2} n q e a_0$



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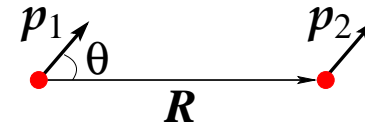
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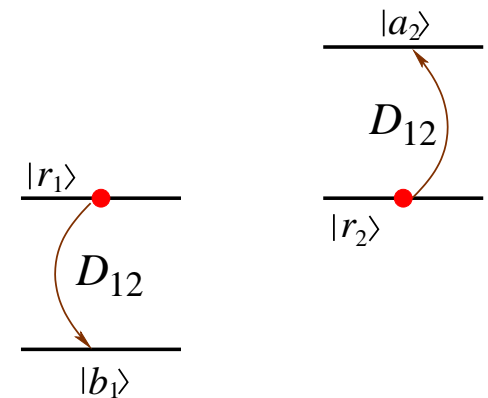
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⇒ Resonant DDI

$$D = \frac{1}{R^3} \left[\rho_{1+} \rho_{2-} + \rho_{1-} \rho_{2+} + \rho_{1z} \rho_{2z} (1 - 3 \cos^2 \theta) \right. \\ \left. - \frac{3}{2} \sin^2 \theta (\rho_{1+} \rho_{2+} + \rho_{1+} \rho_{2-} + \rho_{1-} \rho_{2+} + \rho_{1-} \rho_{2-}) \right. \\ \left. - \frac{3}{\sqrt{2}} \sin \theta \cos \theta (\rho_{1+} \rho_{2z} + \rho_{1-} \rho_{2z} + \rho_{1z} \rho_{2+} + \rho_{1z} \rho_{2-}) \right]$$

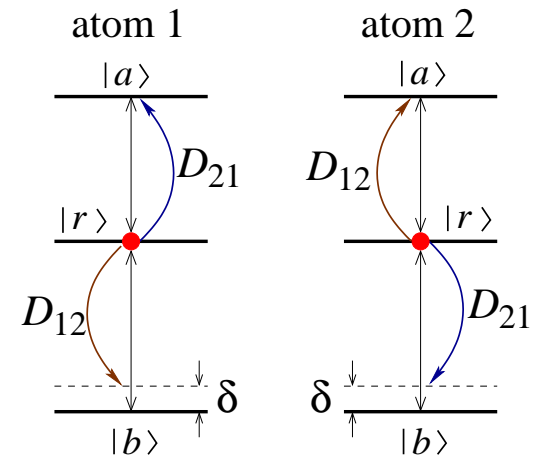


van der Waals Interaction



RDDI (Förster process)

$$D_{12} \equiv D(R) \propto \frac{\wp_{br}\wp_{ar}}{R^3} \propto n^4$$



van der Waals Interaction

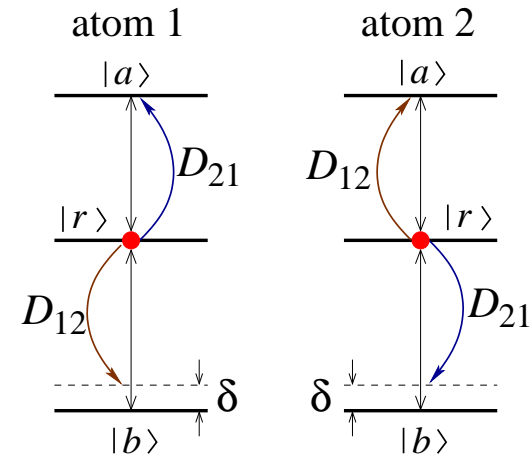


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$$\boxed{\omega_{rb} - \omega_{ar} = \delta \gg D}$$

$$(\delta \propto n^{-3})$$



$\Rightarrow |r_1\rangle |r_2\rangle \rightarrow |a_{1,2}\rangle |b_{2,1}\rangle$: **Non-Resonant DDI** (Adiabatic elim. $|a_{1,2}\rangle |b_{2,1}\rangle$)

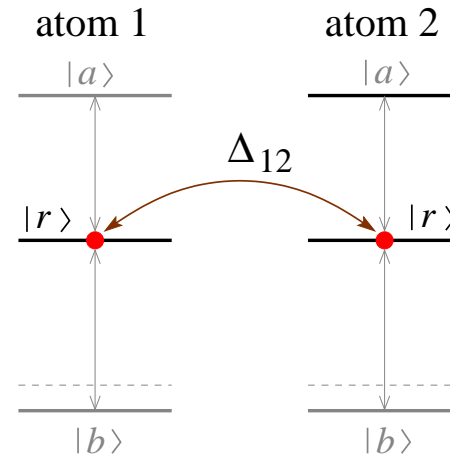
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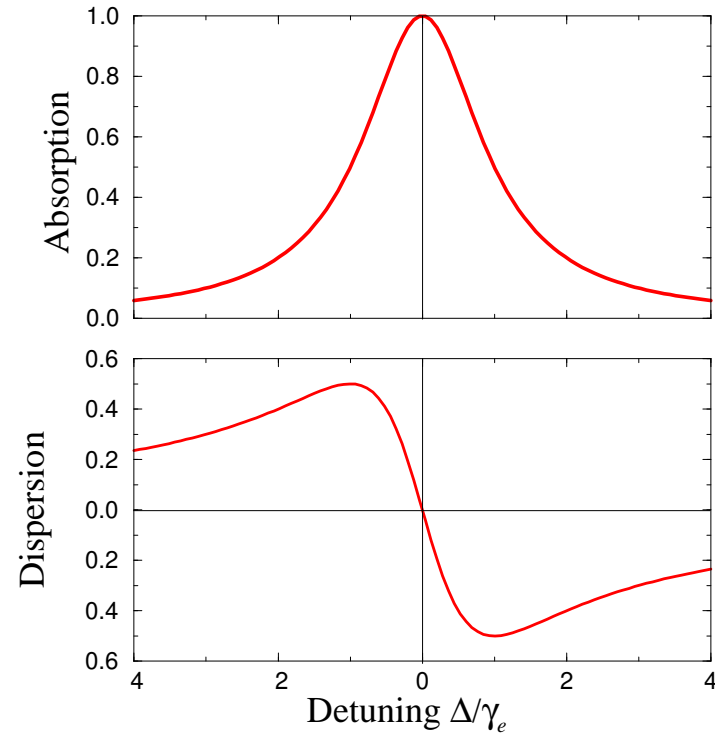
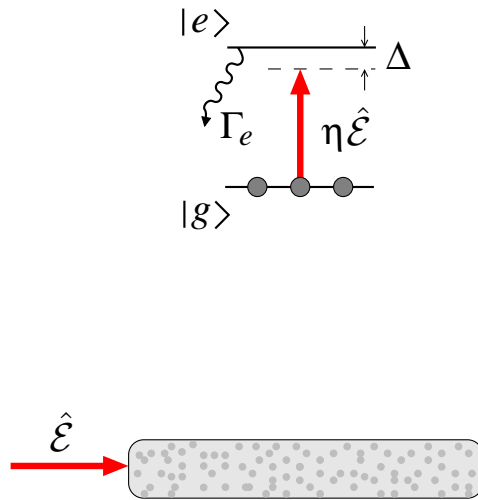
\Rightarrow Energy shift of $|r_1\rangle |r_2\rangle$ (2nd-order in D/δ)

$$\mathcal{V}_{\text{vdW}} = \hbar \hat{\sigma}_{rr}^1 \Delta_{12} \hat{\sigma}_{rr}^2$$

$$\Delta_{12} \equiv \bar{\Delta}(R) = 2 \frac{|D(R)|^2}{\delta} = \frac{C_6}{R^6} \propto n^{11} \text{ — vdWI strength}$$

Electromagnetically Induced Transparency in atomic medium

Electromagnetically Induced Transparency



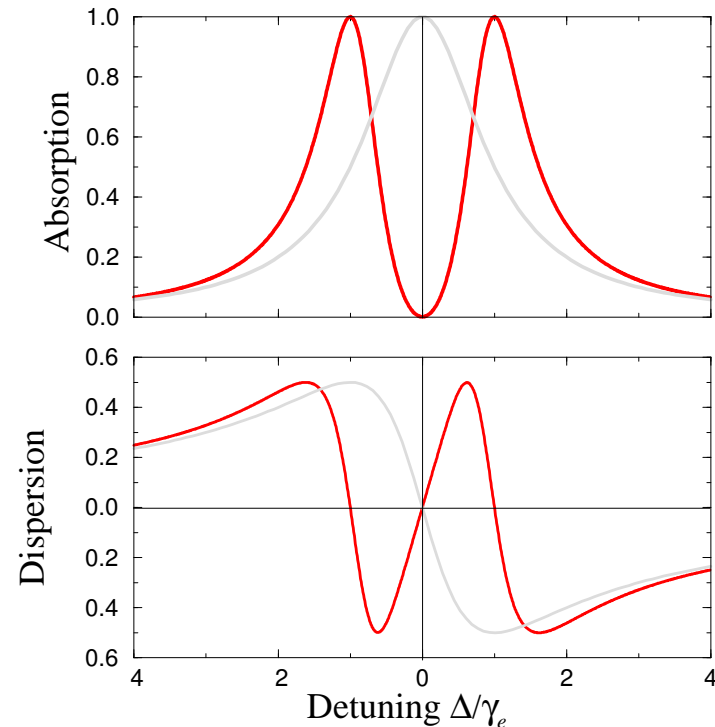
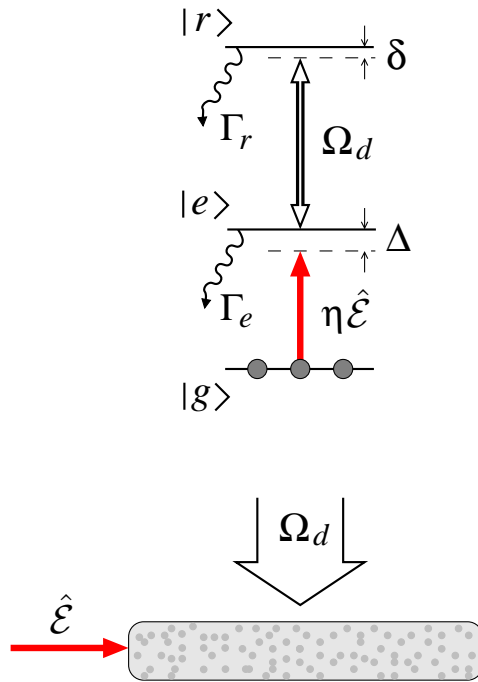
Stationary propagation

$$\partial_z \mathcal{E} = i \frac{\omega}{2c} \chi \mathcal{E}$$

2LA medium susceptibility

$$\chi = \frac{2}{\omega} \frac{i\eta^2 N}{\gamma_e - i\Delta} = \frac{i}{k} \frac{\sigma_0 \bar{\rho} \Gamma_e}{\gamma_e - i\Delta}$$

Electromagnetically Induced Transparency



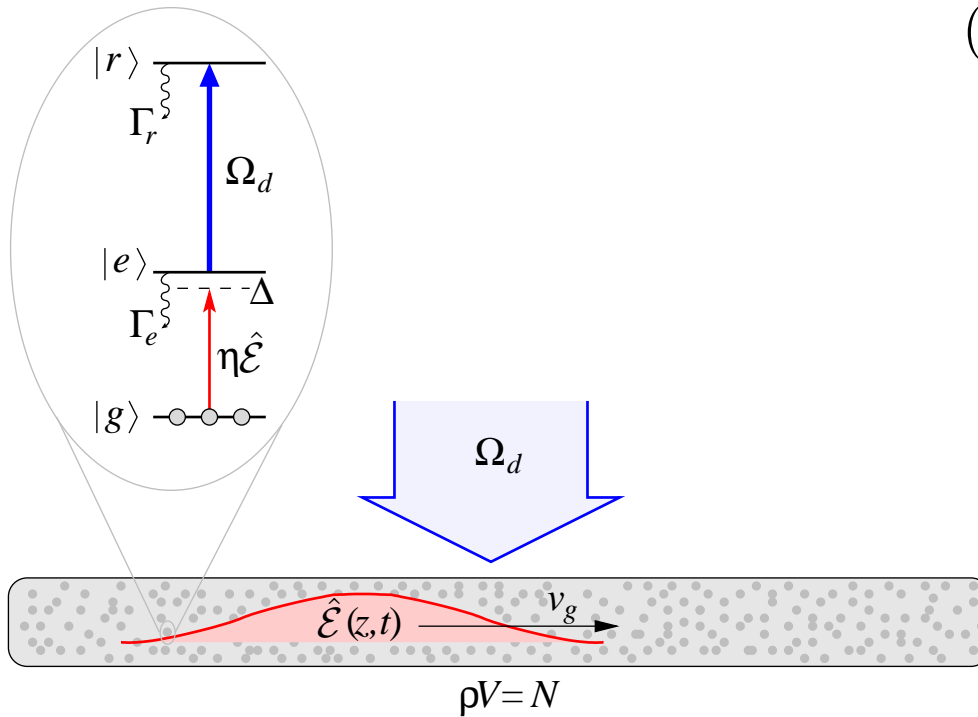
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EIT (3LA) susceptibility

$$\chi = \frac{2}{\omega} \frac{i\eta^2 N}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_r - i(\Delta + \delta)}}$$

Pulse propagation in EIT medium



$$(\partial_t + v_g \partial_z) \hat{\mathcal{E}}(z, t) = i \frac{\omega}{2} \chi \hat{\mathcal{E}}(z, t)$$

with

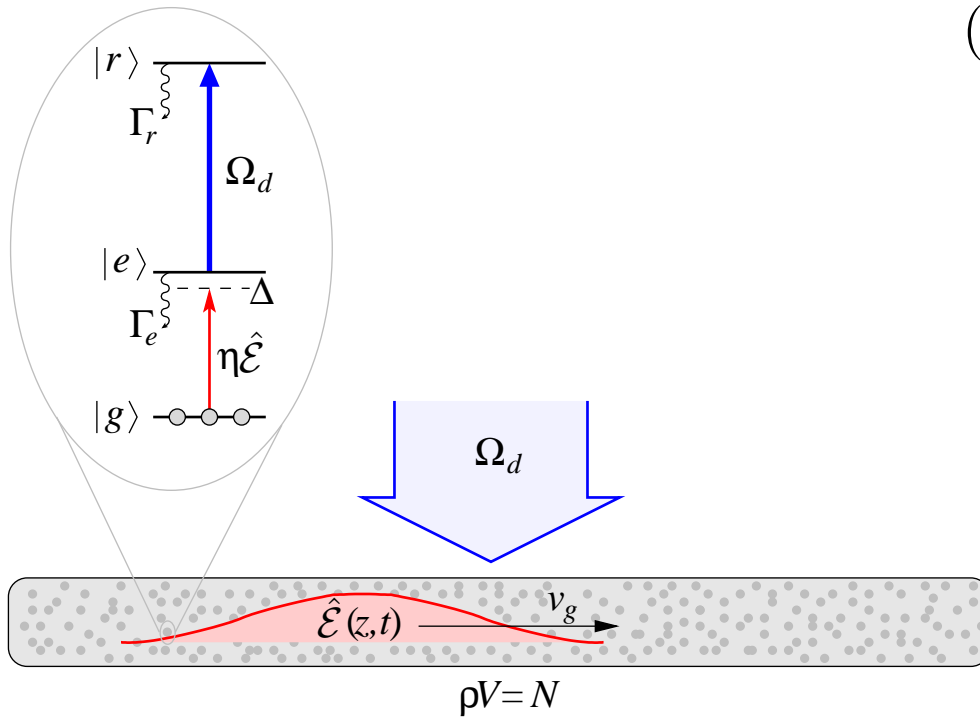
$$v_g = \frac{c}{1 + \frac{\omega}{2} \frac{\partial}{\partial \Delta} \text{Re} \chi} \simeq c \frac{|\Omega_d|^2}{\eta^2 N} \ll c$$

$$\frac{\omega}{2} \chi \simeq \frac{\eta^2 N}{|\Omega_d|^2} (\Delta + \delta) \rightarrow 0$$

@ two-photon resonance

$$\Delta \simeq -\delta \quad \& \quad |\gamma_e + i\delta| \gamma_r \ll |\Omega_d|^2$$

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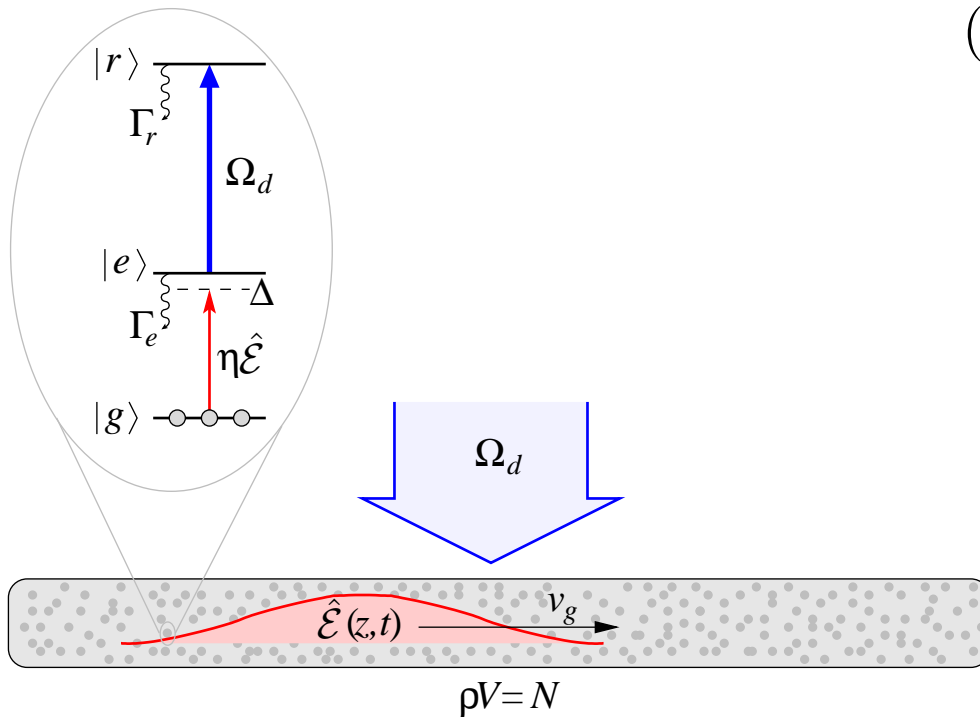
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Inside the medium \Rightarrow dark-state polariton

$$\hat{\mathcal{E}} \rightarrow \hat{\Psi} = \cos \Theta \hat{\mathcal{E}} - \sin \Theta \sqrt{N} \hat{\sigma}_{gr} \quad \text{with} \quad \tan^2 \Theta = \frac{\eta^2 N}{|\Omega_d|^2} \gg 1 \quad \left[\eta = \frac{\wp_{ge}}{\hbar} \sqrt{\frac{\hbar \omega}{2 \epsilon_0 V}} \right]$$

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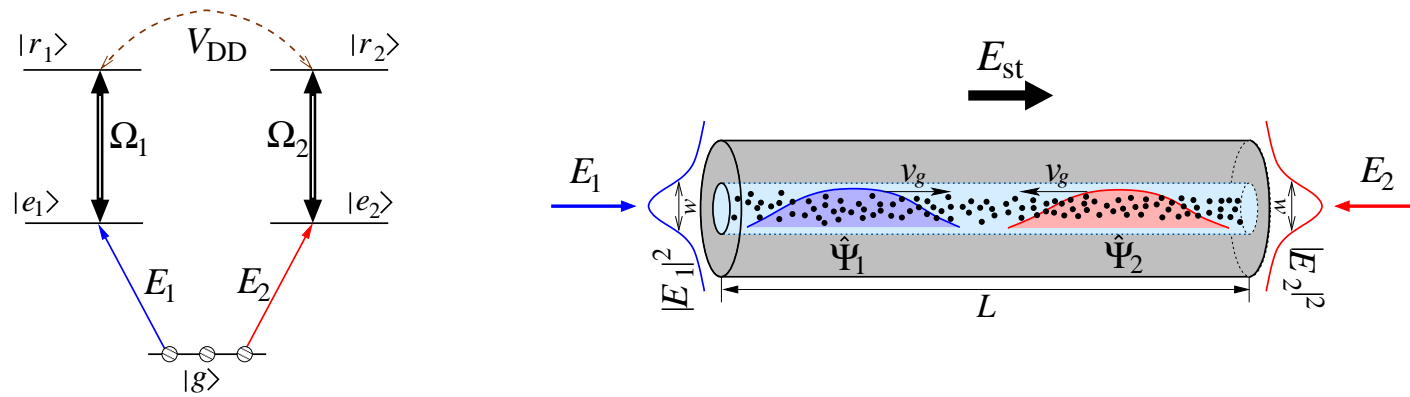
Photons are converted to atomic $|r\rangle$ excitations ($\cos \Theta \ll 1$) and propagate with slow (variable) group velocity $v_g(t) = c \cos^2 \Theta(t) \ll c$:

$$(\partial_t + v_g \partial_z) \hat{\Psi} = 0 \quad \Rightarrow \quad \hat{\Psi}(z, t) = \hat{\Psi}(z - \int_0^t v_g(t') dt', 0)$$

Giant cross-phase modulation: Photonic phase gate

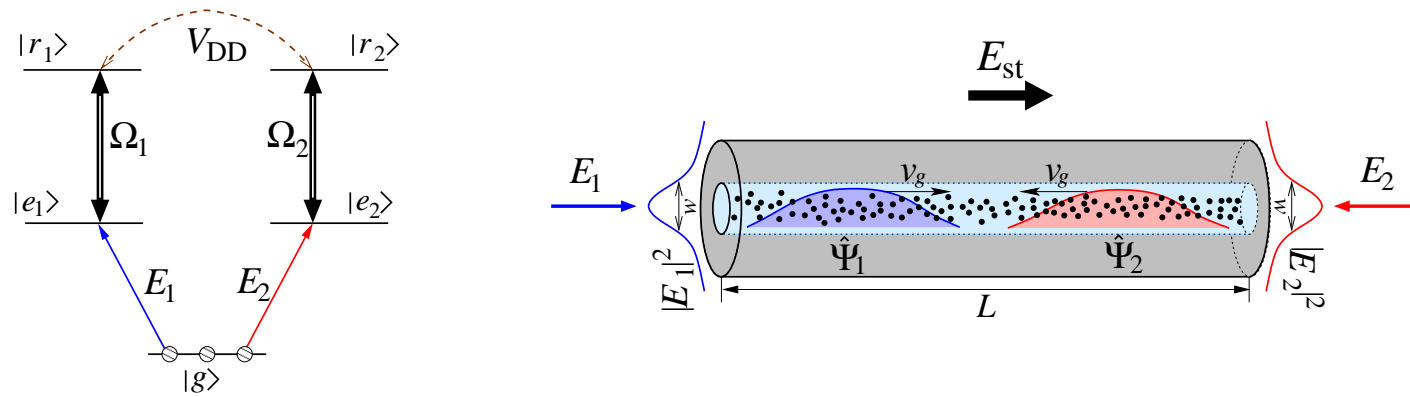
Friedler, Petrosyan, Fleischhauer, Kurizki, PRA **72**, 043803 (2005)
Shahmoon, Kurizki, Fleischhauer, Petrosyan PRA **83**, 033806 (2011)

Photon-Photon Interaction



Static $E_{st} \mathbf{e}_z \Rightarrow$ Stark eigenstates $|r_j\rangle$ with SDMs $\phi_{\partial r} \mathbf{e}_z = \frac{3}{2} n q e a_0 \mathbf{e}_z$

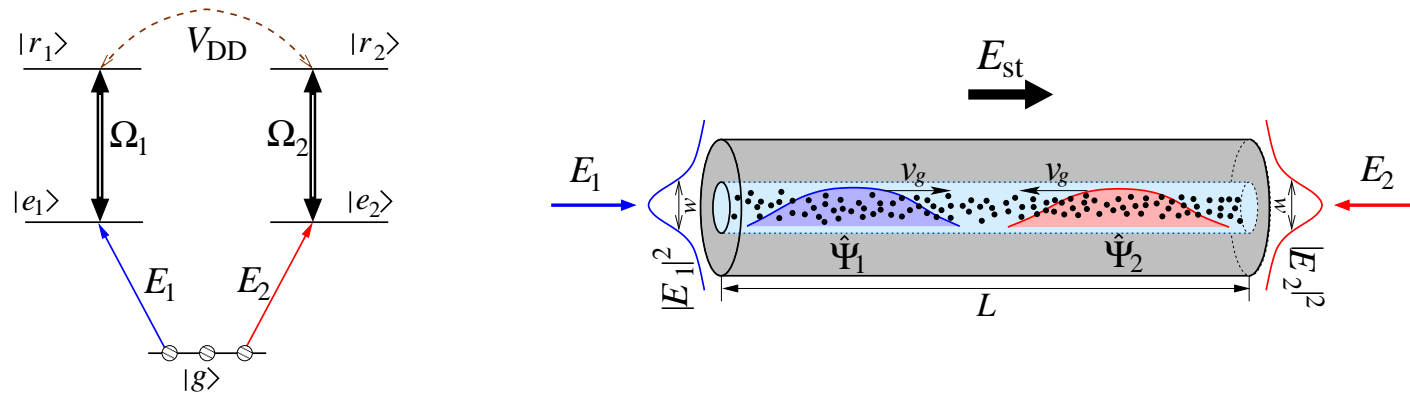
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$E_j \rightarrow \hat{\Psi}_j = \cos \Theta \hat{\mathcal{E}}_j - \sin \Theta \sqrt{N} \hat{\sigma}_{gr_j}$ ($j = 1, 2$) propagate with $\pm v_g = c \cos^2 \Theta$

Photon-Photon Interaction



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Atomic components of $\hat{\Psi}_i$ interact via Static DDI \Rightarrow induces **XPM**

$$V_{DD} = \hbar \rho^2 \iint d^3 r d^3 r' \hat{\sigma}_{rr}(\mathbf{r}) D(\mathbf{r} - \mathbf{r}') \hat{\sigma}_{rr}(\mathbf{r}')$$

$$D(\mathbf{r} - \mathbf{r}') = C \frac{1 - 3 \cos^2 \theta}{|\mathbf{r} - \mathbf{r}'|^3} \quad \text{with} \quad C = \frac{\wp_{r1} \wp_{r1'}}{4\pi \epsilon_0 \hbar}$$

Resonant DDI (state mixing) is suppressed for $q = n - 1, m = 0$

Cross-Phase Modulation



$$(\partial_t + v_g \partial_z) \hat{\Psi}_j(z, t) = -i \sin^2 \Theta \hat{S}(z, t) \hat{\Psi}_j(z, t)$$

$$\langle \hat{S}(z) \rangle < \delta\omega \simeq \frac{|\Omega_d|^2}{\gamma_e}$$

$$\hat{S}(z, t) = \frac{1}{L} \int_0^L D(z - z') \sin^2 \Theta [\hat{\mathcal{I}}_1(z', t) + \hat{\mathcal{I}}_2(z', t)] dz' \quad \hat{\mathcal{I}}_j \equiv \hat{\Psi}_j^\dagger \hat{\Psi}_j$$

$$D(z - z') = \frac{1}{(\pi w^2)^2} \int d^2 r_\perp \int d^2 r'_\perp e^{-(r_\perp^2 + r'^2_\perp)/w^2} D(\mathbf{r} - \mathbf{r}')$$

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Input state: two single photons [$t = 0, z_1 = 0$ & $z_2 = L$]

$$|\Phi_{\text{in}}\rangle = |1_1\rangle |1_2\rangle \quad [|1_j\rangle = \frac{1}{L} \int dz f_j(z) \hat{\Psi}_j^\dagger(z) |0\rangle]$$

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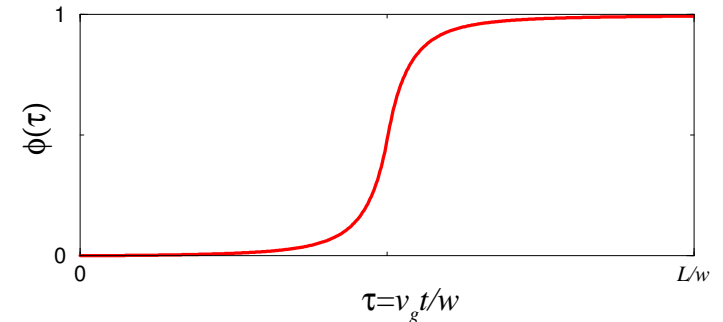
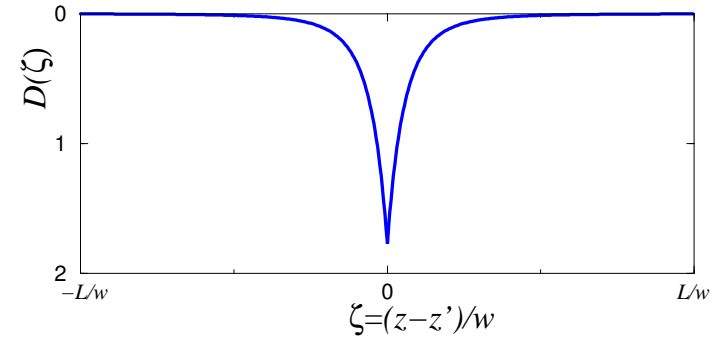
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Evolution of two-photon wavefunction:

$$F_{12}(z_1, z_2, t) = \langle 0 | \hat{\Psi}_1(z_1, t) \hat{\Psi}_2(z_2, t) | \Phi_{\text{in}} \rangle$$

$$= f_1(z_1 - vt) f_2(z_2 + vt) e^{i\phi(z_1, z_2, t)}$$

$$\phi(z_1, z_2, t) = -\sin^4 \Theta \int_0^t dt' D(z_1 - z_2 - 2v_g(t - t'))$$



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Output state: [$t = L/v_g, z_1 = L$ & $z_2 = 0$]

$$|\Phi_{\text{out}}\rangle = e^{i\phi} |1_1\rangle |1_2\rangle$$

$$\phi(L, 0, L/v) = -\frac{\sin^4 \Theta}{v_g} \int_0^L dz' D(2z' - L) = \frac{C}{v_g w^2}$$

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Phase shift $\phi = \pi$ [spatially uniform!]

\Rightarrow Universal CPHASE gate between SPh pulses $\hat{\mathcal{E}}_1$ & $\hat{\mathcal{E}}_2$

$$|x\rangle_1 |y\rangle_2 \rightarrow (-1)^{xy} |x\rangle_1 |y\rangle_2 \quad (x, y \in [0, 1])$$

Dissipative photonic interactions

Gorshkov et al., PRL **107**, 133602 (2011); Peyronel et al., Nature **488**, 57 (2012)
Pritchard et al., PRL **105**, 193603 (2010); Petrosyan et al., PRL **107**, 213601 (2011)
Ates et al. PRA **83**, 041802(R) (2011); Sevincli et al. PRL **107**, 153001 (2011)
Baur et al. PRL **112**, 073901 (2014); Gorniaczyk et al. PRL **113**, 053601 (2014)

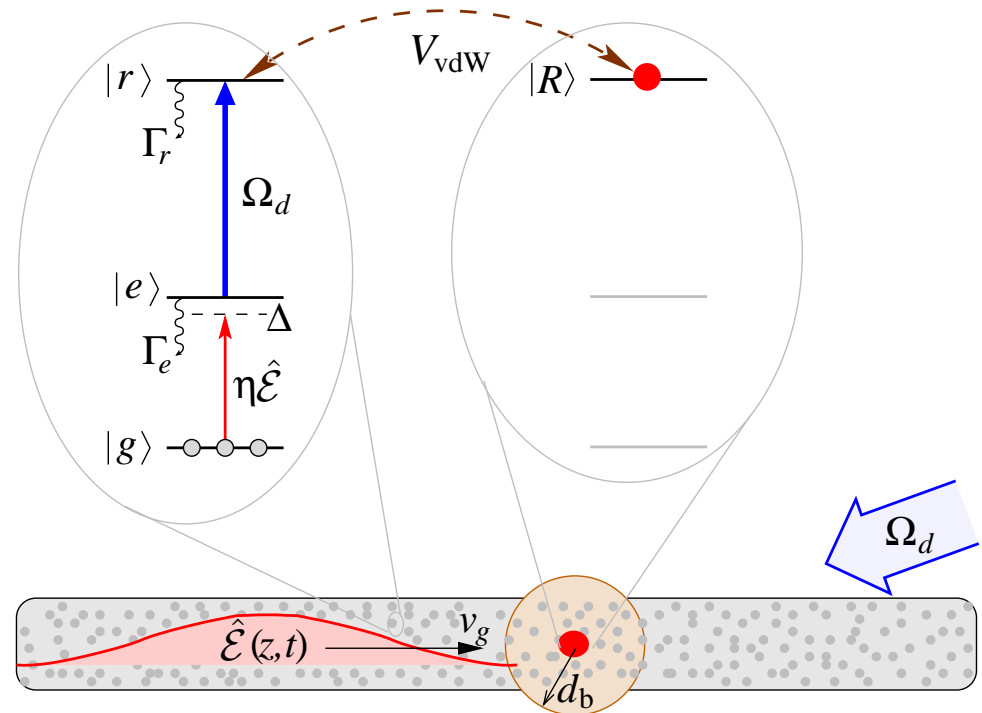
Rydberg blockade of excitation

Atom in state $|R\rangle$ at position \mathbf{r}'
interacts with the medium
atoms via

$$V_{\text{vdW}} = \sum_j \hat{\sigma}_{rr}^j \bar{\Delta}(\mathbf{r}_j - \mathbf{r}') \hat{\sigma}_{RR}$$

$$\bar{\Delta}(\mathbf{r}_j - \mathbf{r}') = \frac{C_6}{|\mathbf{r}_j - \mathbf{r}'|^6}$$

level shift of $|r\rangle$



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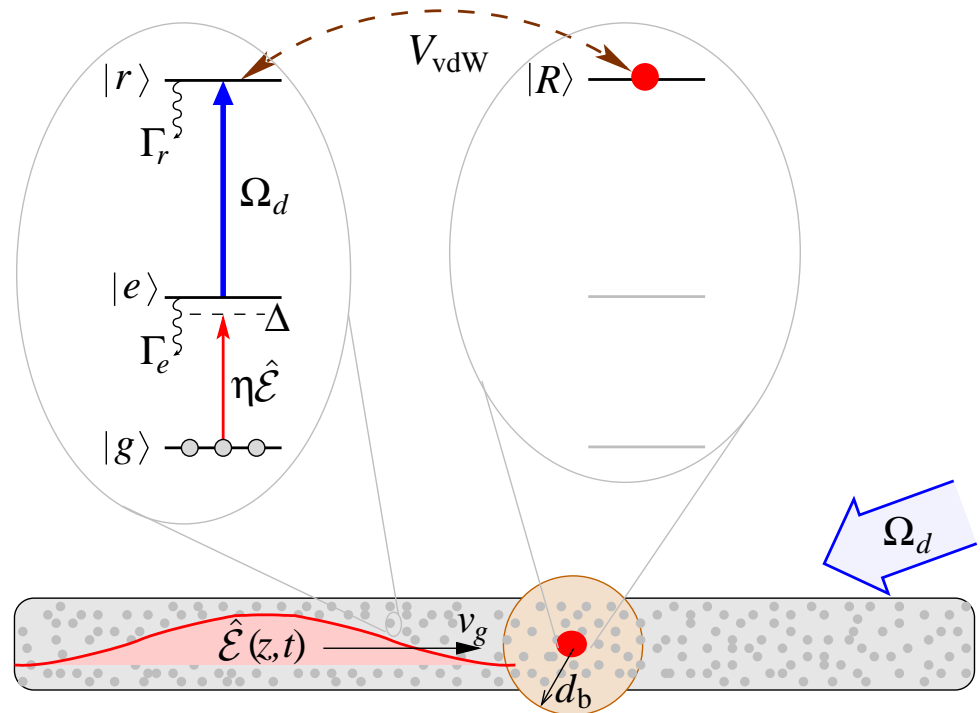
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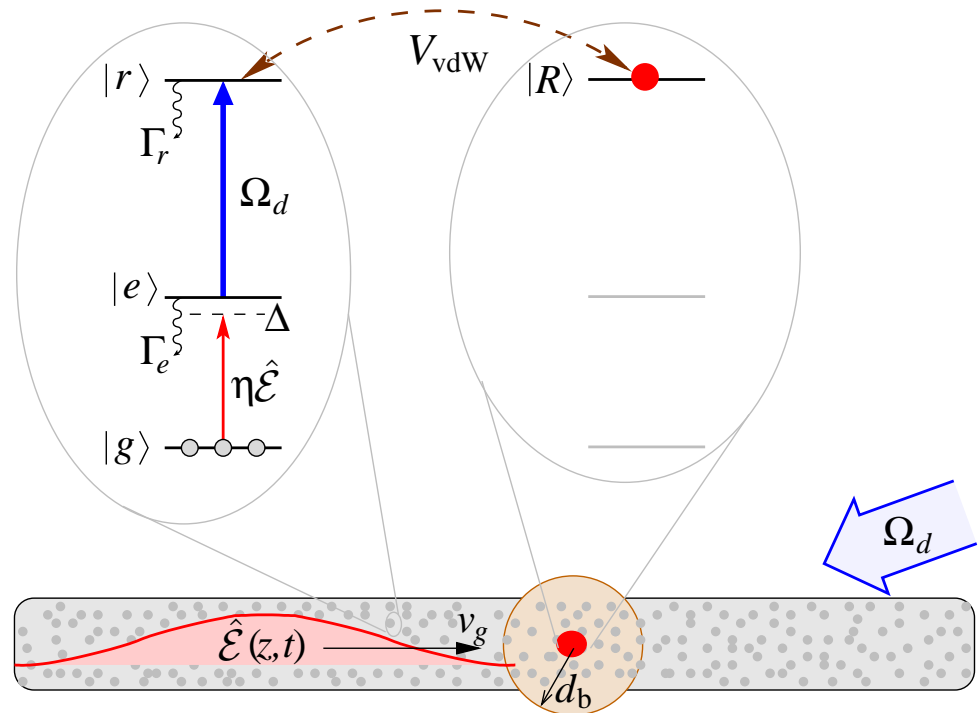
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1D ($w < d_b$) propagation $\partial_z \mathcal{E} = i \frac{\omega}{2c} \chi \mathcal{E}$ with susceptibility

$$\chi(z) = \frac{2}{\omega} \frac{i\eta^2 N}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_r - i(\Delta - \bar{\Delta}(z-z'))}} = \begin{cases} \frac{i}{k} \frac{\sigma_0 \bar{\rho} \Gamma_e}{\gamma_e - i\Delta + \frac{|\Omega_d|^2}{\gamma_r - i\Delta}} & \text{if } (z - z') > d_b \text{ (EIT)} \\ \frac{i}{k} \frac{\sigma_0 \bar{\rho} \Gamma_e}{\gamma_e - i\Delta} & \text{if } (z - z') \leq d_b \text{ (2LA)} \end{cases}$$



Photon transistor



Stored in $|R\rangle$ (photonic) excitation blocks the transmission of probe field

\Rightarrow photons are (conditionally) absorbed in the blockade region by $\epsilon \simeq e^{-\text{OD}_b}$

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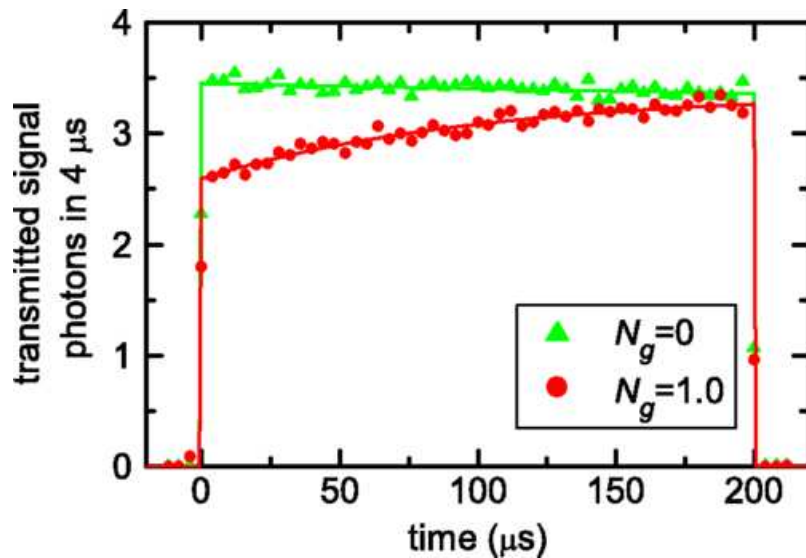
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PRL 113, 053602 (2014) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2014

Single-Photon Transistor Using a Förster Resonance

Daniel Tiarks, Simon Baur, Katharina Schneider, Stephan Dürr, and Gerhard Rempe
Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany



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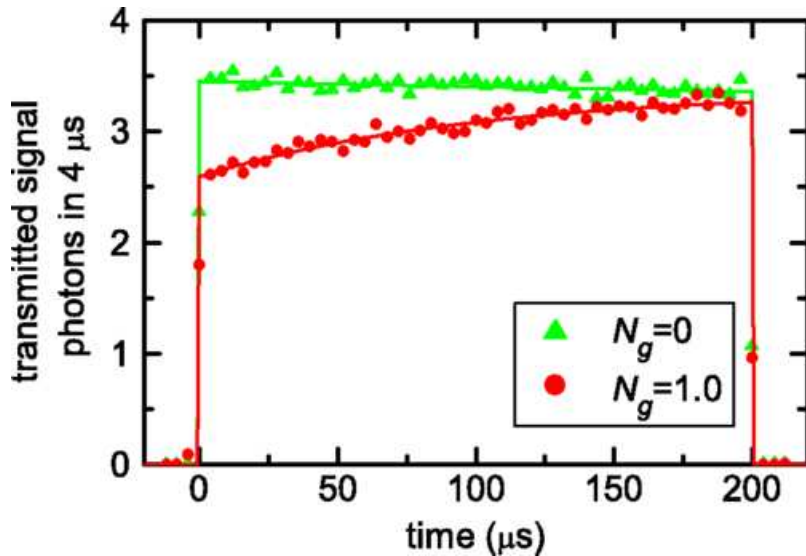
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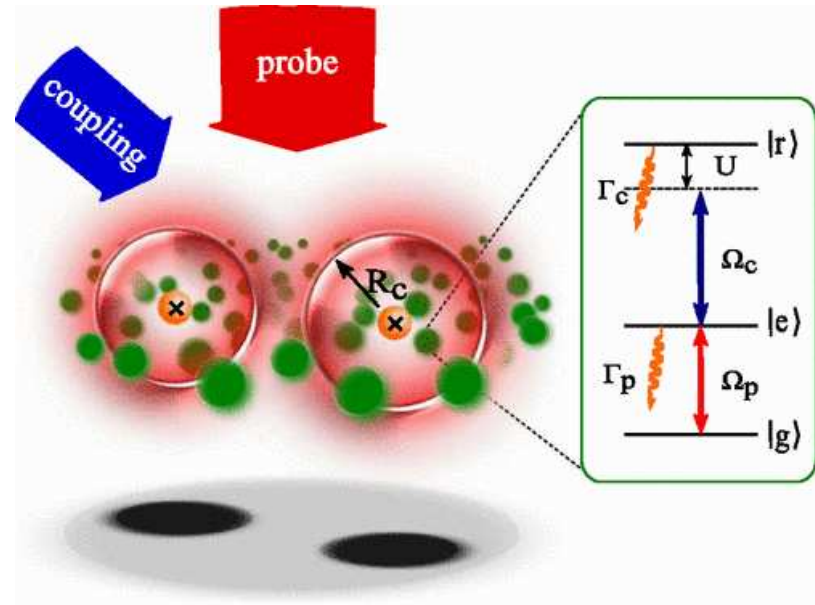
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PRL 108, 013002 (2012) PHYSICAL REVIEW LETTERS 6 JANUARY 2012

Interaction Enhanced Imaging of Individual Rydberg Atoms in Dense Gases

G. Günter, M. Robert-de-Saint-Vincent, H. Schempp, C. S. Hofmann, S. Whitlock,* and M. Weidemüller†
 Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany

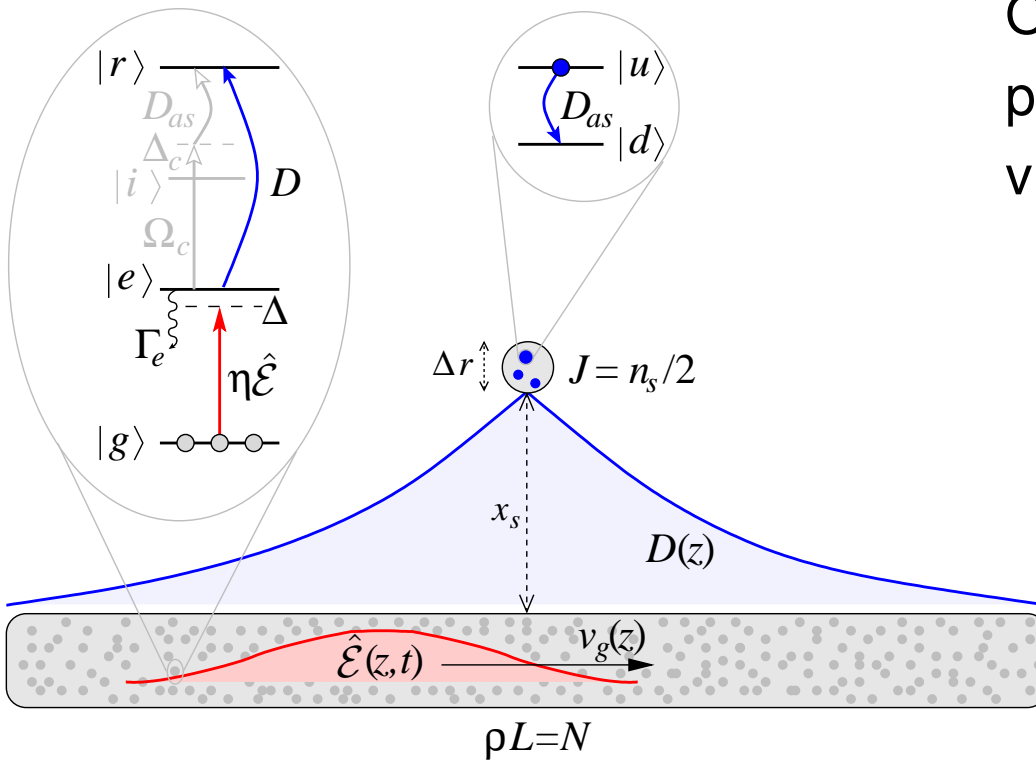




Dipolar Exchange Induced Transparency with Rydberg atoms

Dipolar-exchange induced transparency

One or more spin atoms play the role of Ω_d via exchange interaction



$$D(z) \hat{\sigma}_{re}(z) \otimes \hat{\sigma}_-$$

$$D(z) = \frac{C_3 \Omega_c / \Delta_c}{|z \mathbf{e}_z - \mathbf{r}_j|^3}$$

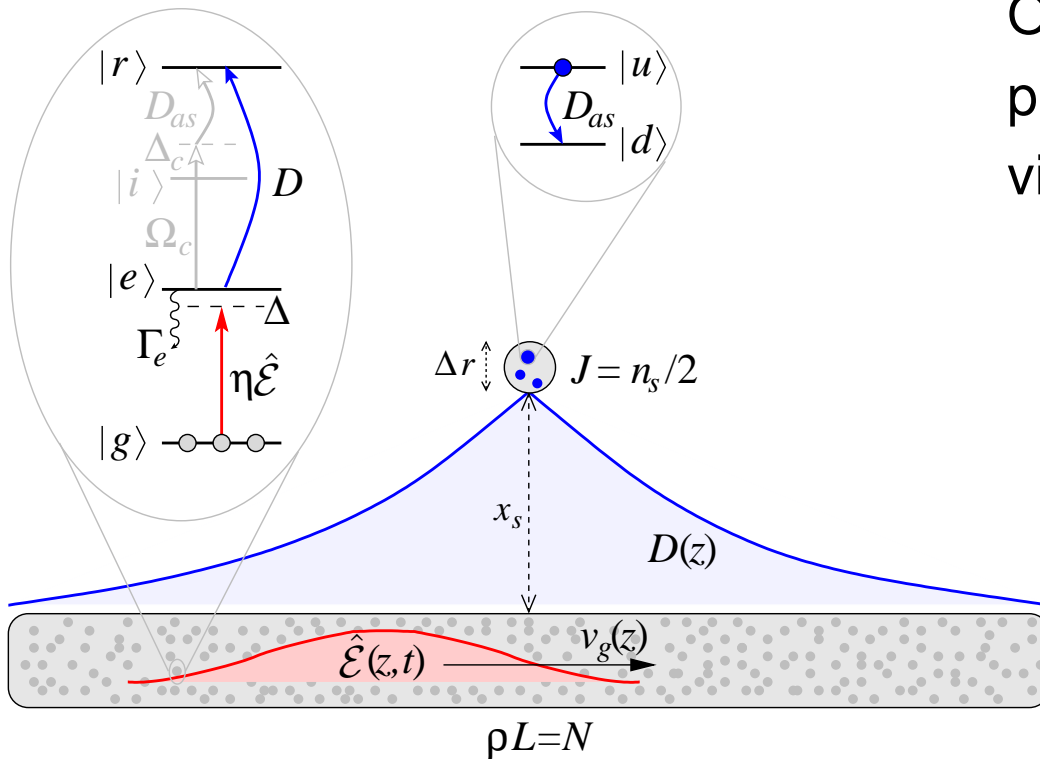
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Each probe photon in the medium attempts to create atomic $|r\rangle$ excitation with *simultaneous* flip of one spin atom $|u\rangle \rightarrow |d\rangle$ [$\hat{J}_- \equiv \sum_j^{n_s} \hat{\sigma}_-^j$ ($\Delta r \ll x_s$)]

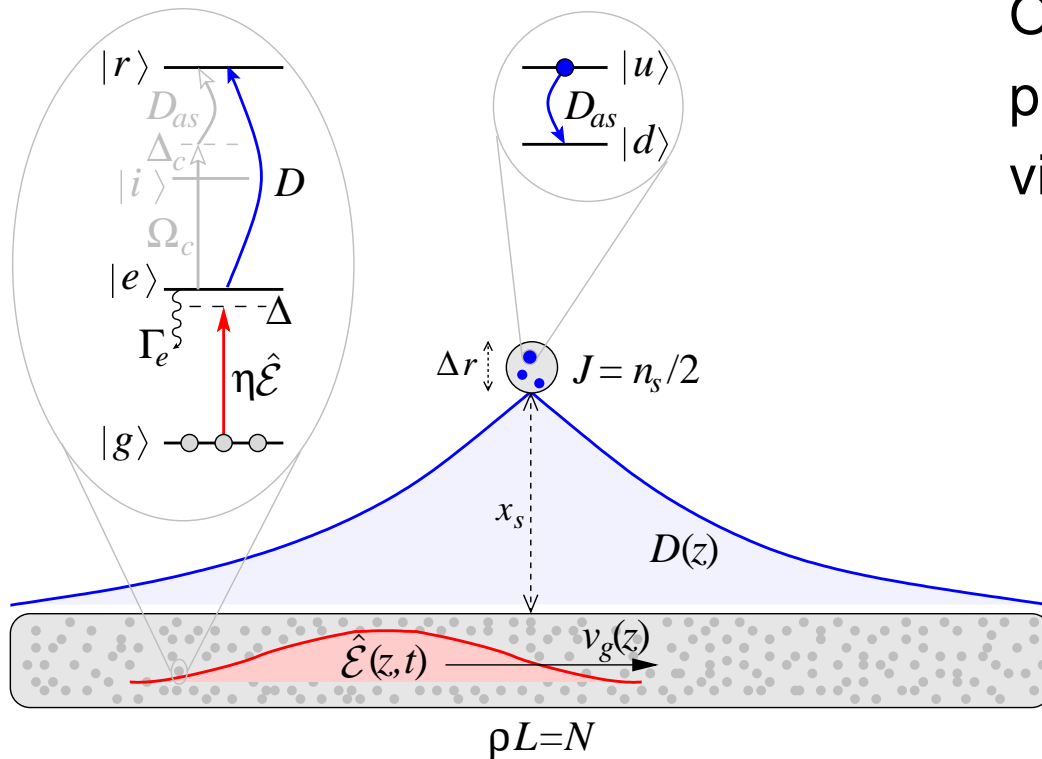
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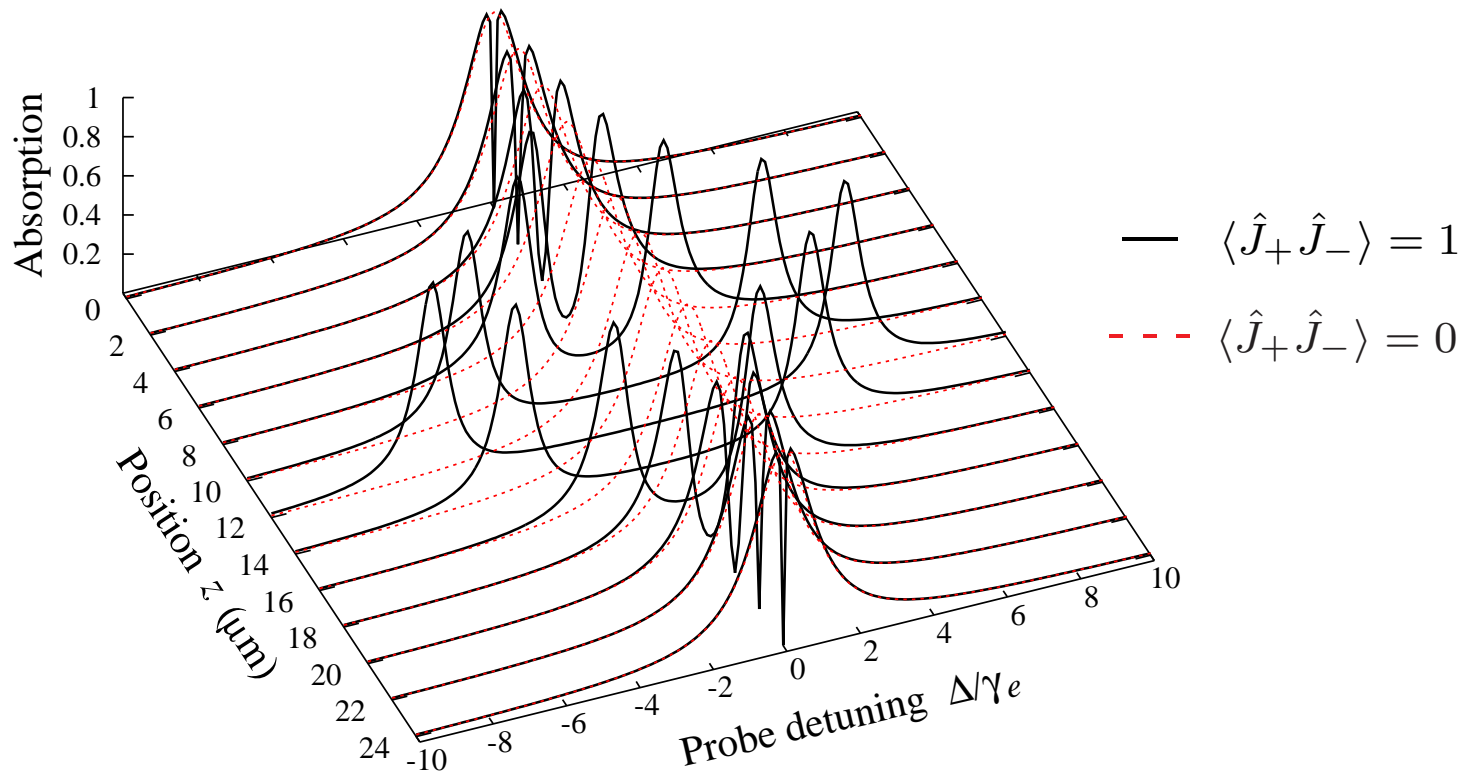


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\Rightarrow DEIT exists for $n_p \leq n_s \equiv 2J$ photons ["spin" state $|J, J - n_p\rangle$]

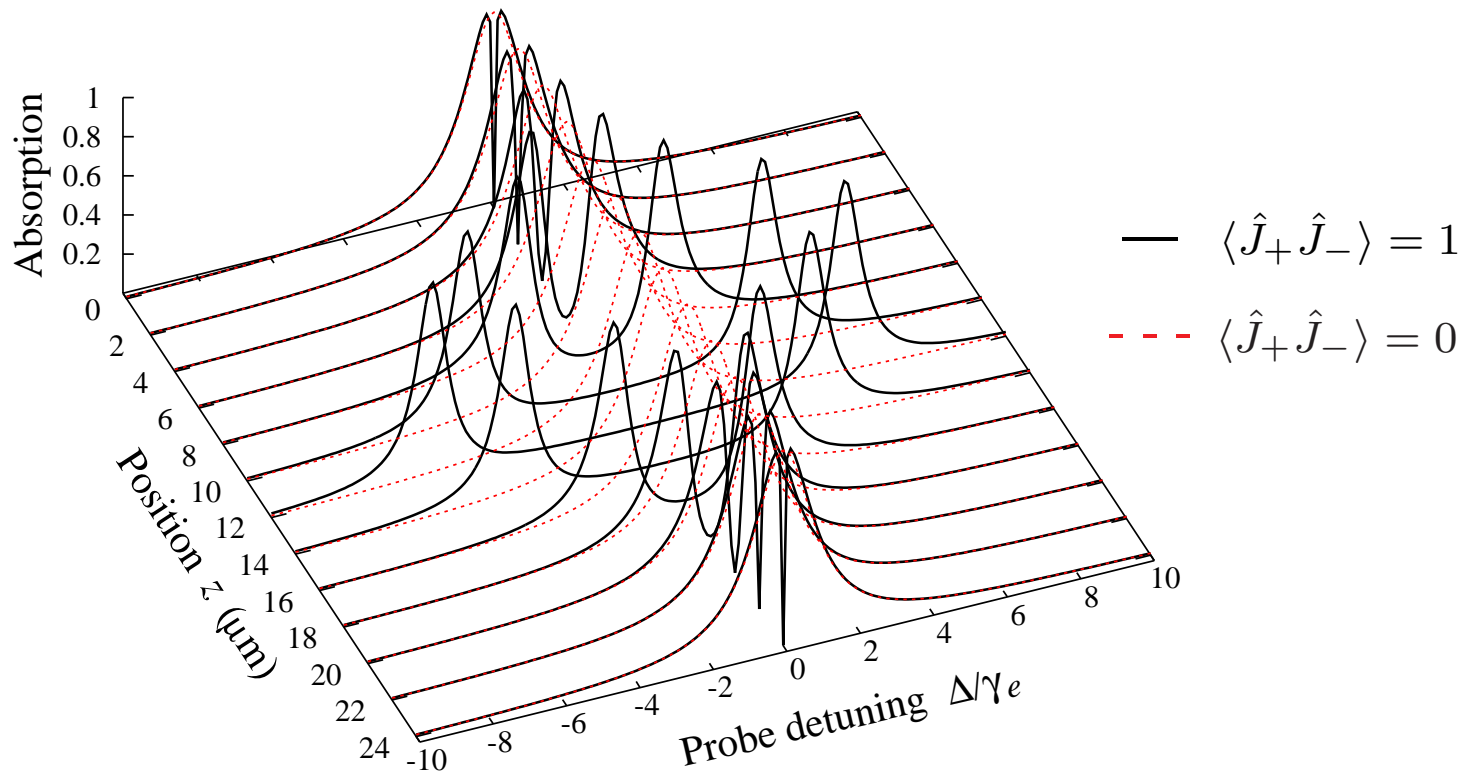
\Rightarrow $(n_p - n_s)$ photons are absorbed by resonant 2LA medium: $OD = 2\sigma_0 \bar{\rho} L > 1$

DEIT medium response



$$\text{Susceptibility } \hat{\chi}(z, \Delta) = \frac{2}{\omega} \frac{i\eta^2 N}{\gamma_e - i\Delta + \frac{|D(z)|^2 \hat{J}_+ \hat{J}_-}{\gamma_r - i(\Delta + \delta)}} \Rightarrow n_p\text{-dependent}$$

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group velocity @ $\Delta \simeq -\delta$

$\hat{v}_g(z) \simeq c \frac{|D(z)|^2 \hat{J}_+ \hat{J}_-}{\eta^2 N} \Rightarrow v_g^{(n_p+1)}(z) = c \frac{|D(z)|^2}{\eta^2 N} (n_s - n_p)(n_p + 1)$

Conclusions



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Reviews:

Murray & Pohl, Adv. Atom. Mol. Opt. Phys. **65**, 321 (2016)

Firstenberg, Adams, Hofferberth, J. Phys. B **49**, 152003 (2016)