Hybrid Quantum Computers

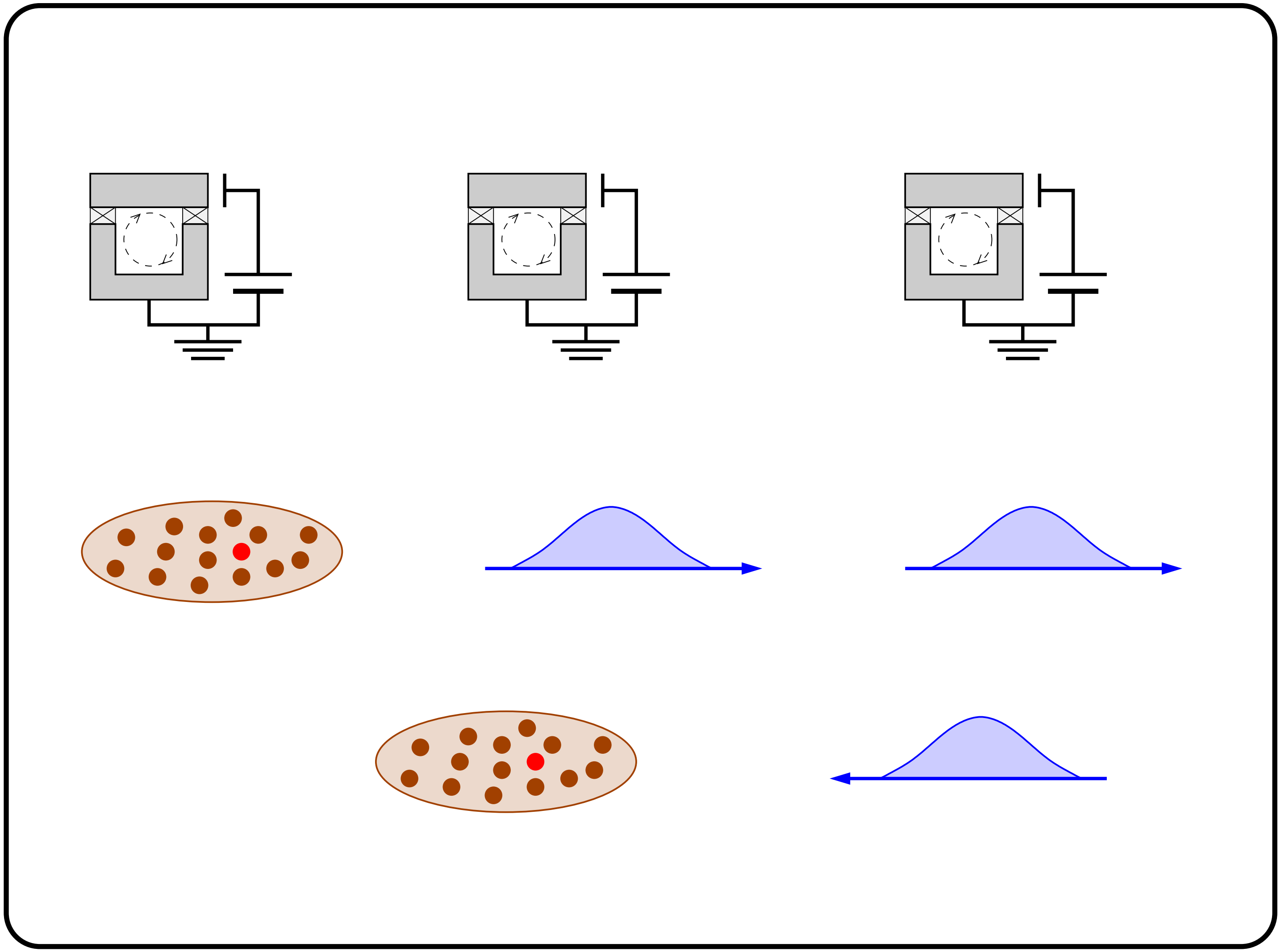
with superconducting circuits

& atomic ensembles

David Petrosyan

IESL-FORTH, Greece

Hybrid Quantum Processor



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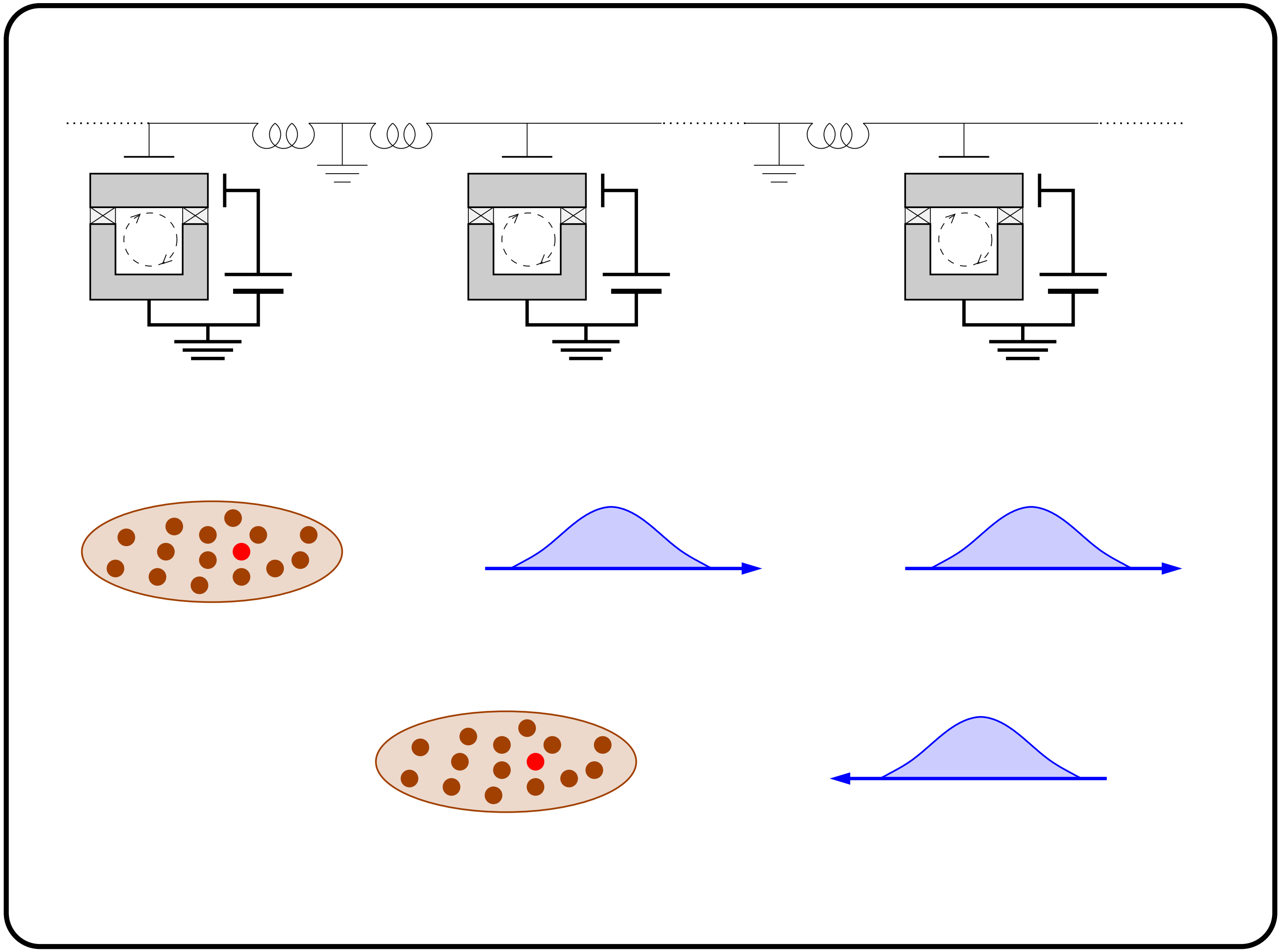

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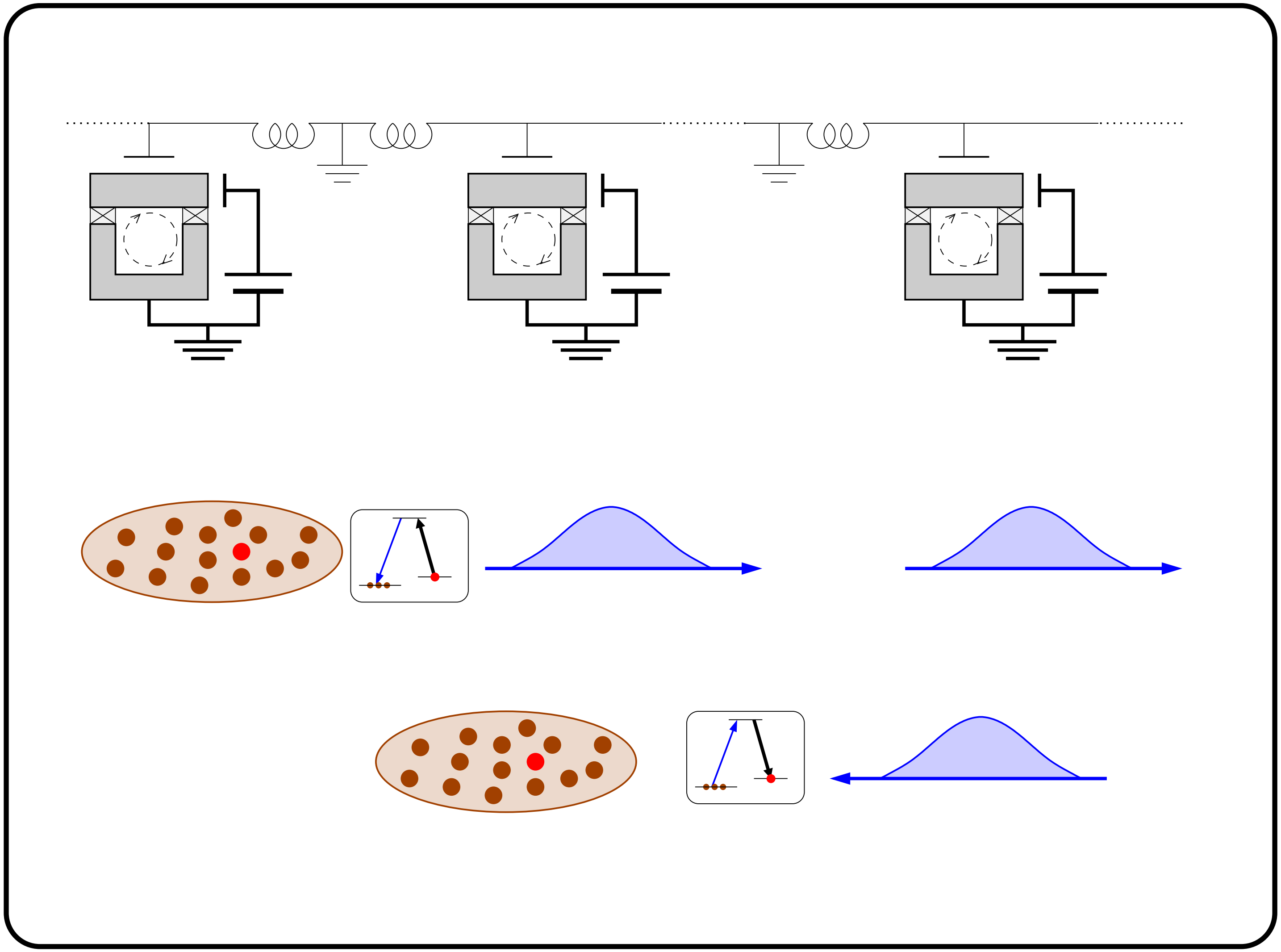

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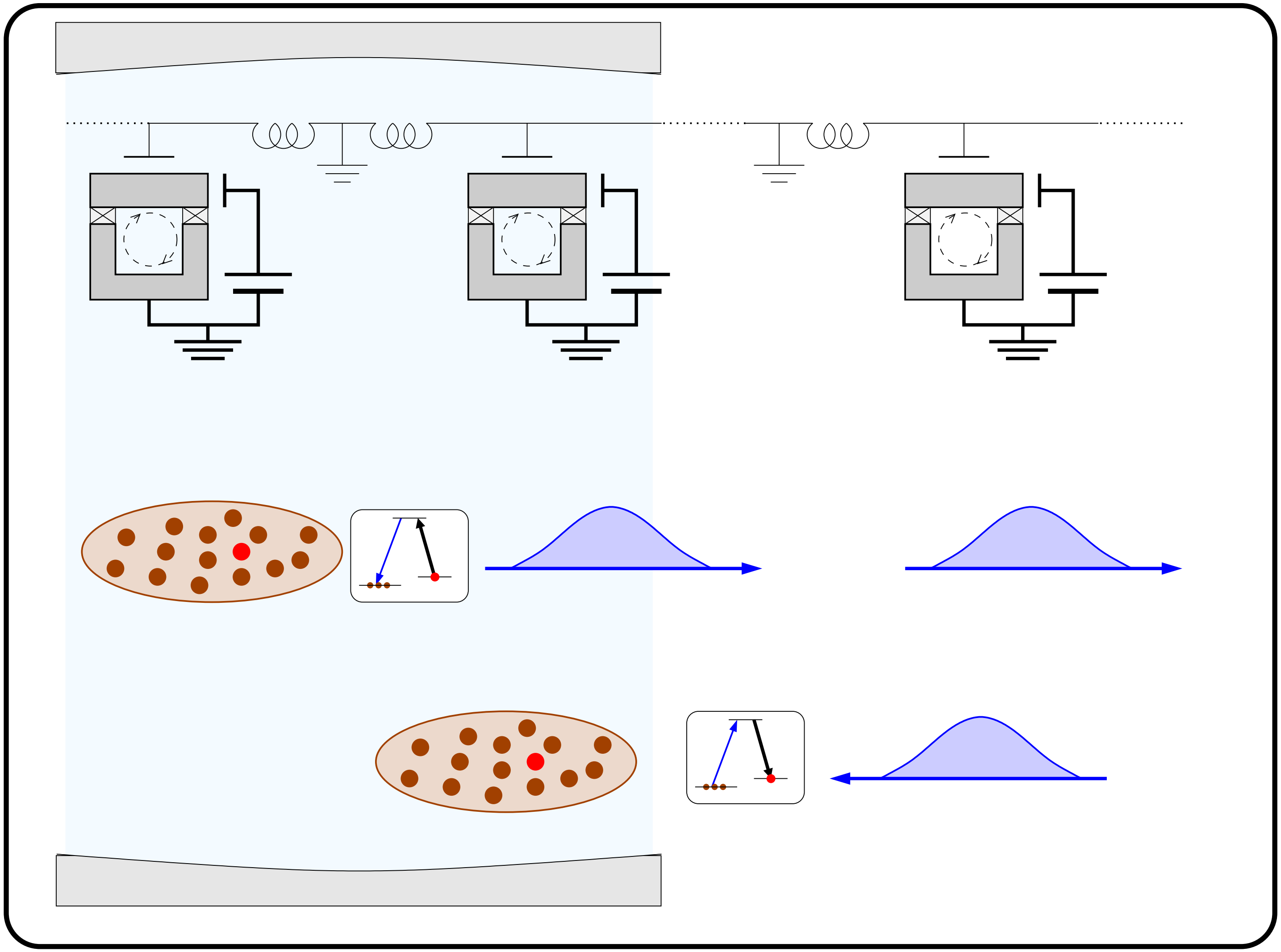

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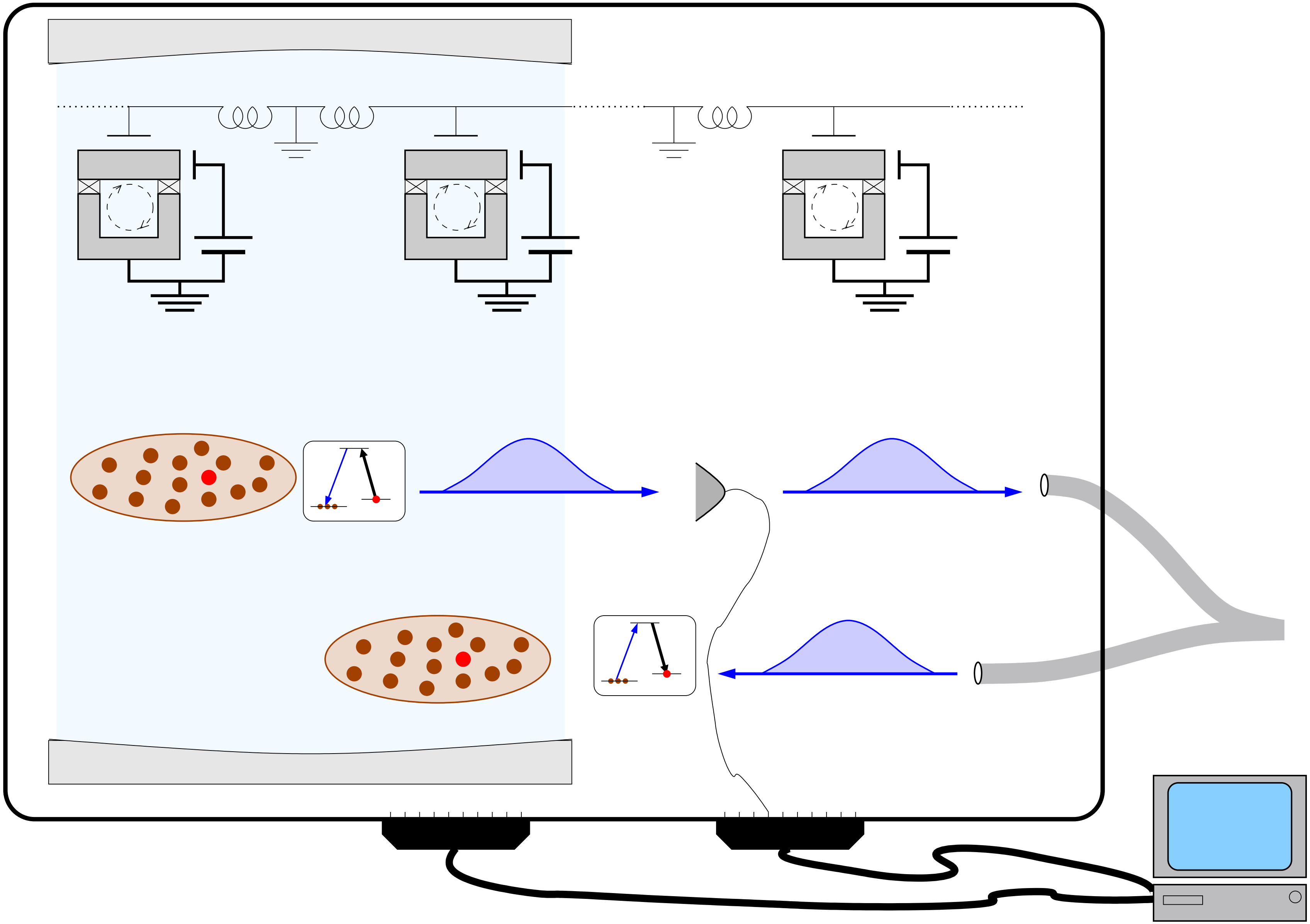

Outline

- Superconducting (Processing) Qubits

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  - *Filtering Inhomogeneously Broadened SE*

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- Atom-Atom Interactions in CPW Resonator
  - *Intracavity resonant dipole-dipole interaction*
  - *Van der Waals interaction*
  - *Applications: Interface of SPh and AE qubits*

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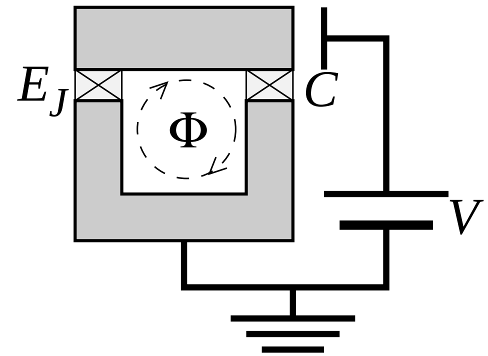
  - *Applications: Interface of SPh and AE qubits*
- Conclusions

Superconducting Charge Qubit

Cooper-pair box Hamiltonian



$$H = 4E_C \sum_n (n - \bar{n})^2 \, |n\rangle\langle n|$$
$$\qquad - E_J \cos(\pi\Phi/\Phi_0) \sum_n (\, |n\rangle\langle n+1| \, + \mathrm{H.c.})$$

$E_C = e^2/2C_{\mathrm{tot}}$ – charging energy

$\bar{n} = CV/2e$ – injected polarization charge

E_J – Josephson tunneling energy ($E_J/\hbar \sim 10 - 20$ GHz)

$\Phi = AB_\perp$ – external magnetic flux ($\Phi_0 = hc/2e$ flux quantum)

Makhlin et al., RMP **73**, 357 (2001); Clarke, Wilhelm, Nature **453**, 1031 (2008)

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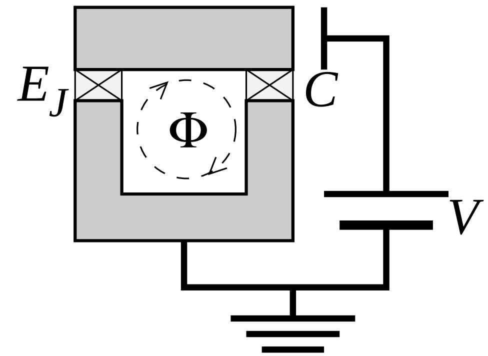

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- For $E_C \gg E_J$ and charge degeneracy point $\bar{n} = \frac{1}{2} \Rightarrow$ TLS

$$|0\rangle = \frac{1}{\sqrt{2}}(\, |0\rangle - \, |1\rangle) \quad |1\rangle = \frac{1}{\sqrt{2}}(\, |0\rangle + \, |1\rangle)$$

with $\hbar\omega_{10} = 2E_J \cos(\pi\Phi/\Phi_0)$ tunable by $B_\perp$

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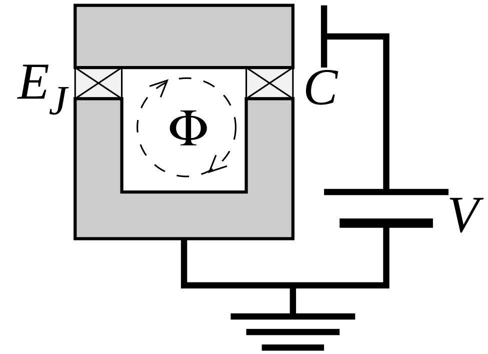

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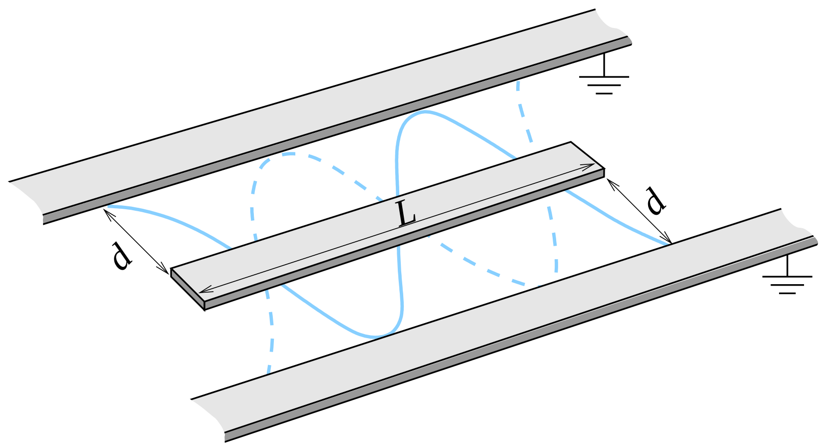
Single-qubit rotations implemented with resonant MW fields ($\wp_{01} \sim 10^4 a_0 e$)

But $1/T_1 = \gamma_q \propto \omega_{10}^3 |\wp_{01}|^2 \gtrsim 10^6$ Hz $\Rightarrow$ fast qubit decoherence

Makhlin et al., RMP **73**, 357 (2001); Clarke, Wilhelm, Nature **453**, 1031 (2008)

Quantum “bus”: CPW resonator



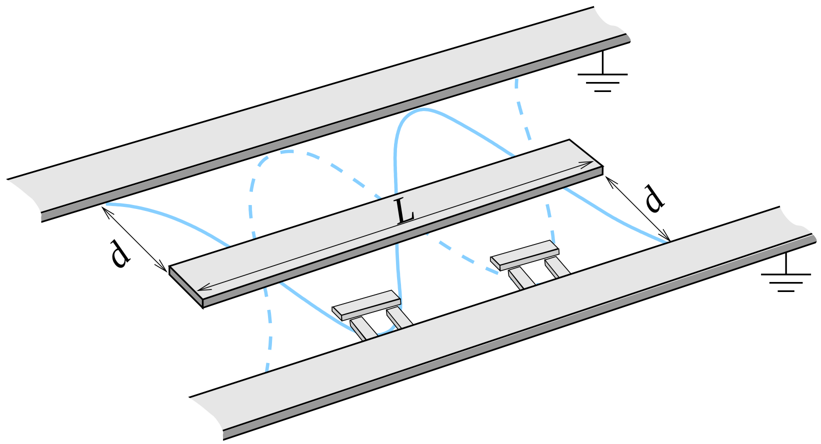
Strip-line length $L \sim 1$ cm and electrode distance $d \sim 3 - 10$ μm

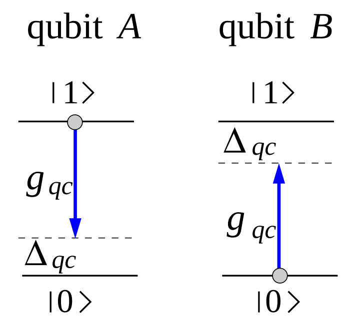
$\omega_c/2\pi = \frac{c}{\lambda_c \sqrt{\epsilon_r}} \sim 3 - 15$ GHz resonant frequencies

$\varepsilon_c = \sqrt{\frac{\hbar\omega_c}{\epsilon_0 \pi d^2 L}} \sim 2 - 5$ mV cm^{-1} field per photon in the cavity $[u(\mathbf{r}) \propto \varepsilon_c e^{-r_\perp/d}]$

$Q \simeq 10^5 - 10^6 \Rightarrow$ photon decay rate $\kappa = \frac{\omega_c}{Q} \simeq 2\pi \times 10^3 - 10^4$ Hz

Coupling SC Qubits via CPW resonator





$g_{qc} = \frac{\wp_{01}}{\hbar}\varepsilon_c \simeq 2\pi \times 50$ MHz qubit–cavity coupling rate; $\Delta_{qc} = \omega_{10} - \omega_c$ detuning

Blais et al., PRA **69**, 062320 (2004); Majer et al., Nature **449**, 443 (2007)

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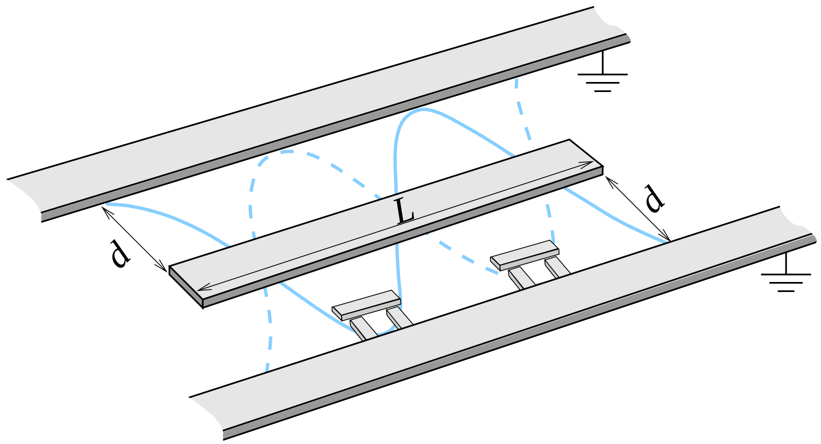

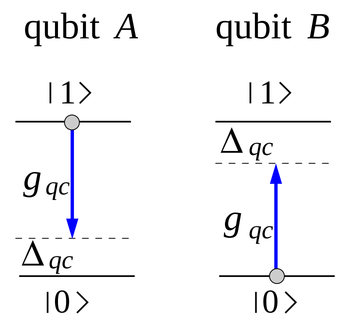

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$\boxed{\Delta_{qc}^{A} = \Delta_{qc}^{B} \gg g_{qc}}$ $\Rightarrow$ **Effective Hamiltonian** (2nd-order in g_{qc}):

$$V_{AB}^{(2)} = \hbar G_{AB}(\hat{\sigma}_+^A \hat{\sigma}_-^B + \hat{\sigma}_+^B \hat{\sigma}_-^A)$$ with $G_{AB} = \frac{g_{qc}^2}{\Delta_{qc}}$ – Exchange constant

SWAP or $\sqrt{\text{SWAP}}$ entangling operations

Blais et al., PRA **69**, 062320 (2004); Majer et al., Nature **449**, 443 (2007)

Photonic Qubit

Single-photon wavepacket in the polarization state

$$|\psi\rangle = \alpha\,|V\rangle + \beta\,|H\rangle \;\text{ with }\; |\alpha|^2 + |\beta|^2 = 1$$

$|V\rangle \equiv |0\rangle$ & $|H\rangle \equiv |1\rangle$ form the computational basis $\{\,|0\rangle, |1\rangle\}$

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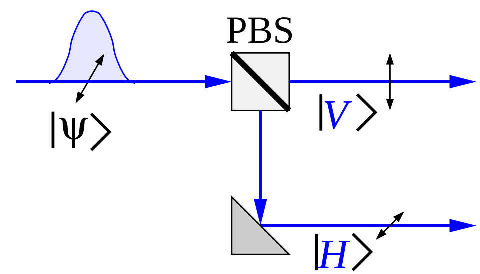
Single-rail $\leftrightarrow$ Dual-rail conversion



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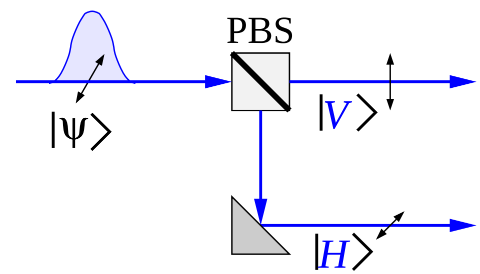
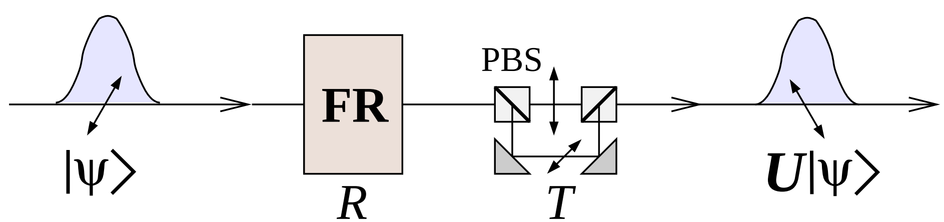
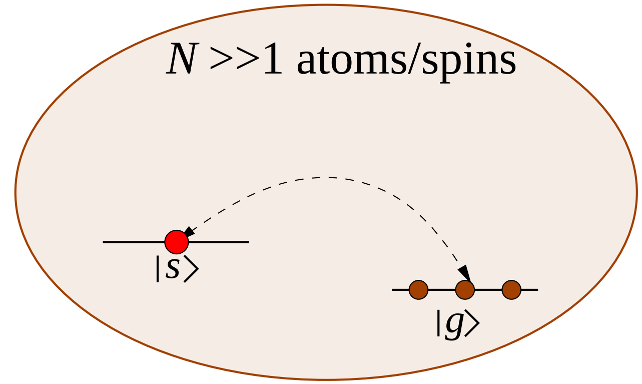


Single-qubit unitary transformations $U \propto R(\vartheta) \otimes T(\phi)$ can be implemented with linear-optics operations



Storage (Ensemble) Qubits



$$|\psi\rangle = \alpha\,|s^{(0)}\rangle + \beta\,|s^{(1)}\rangle$$

with collective states

$$|s^{(0)}\rangle \equiv |g_1, g_2, \ldots, g_N\rangle$$

$$|s^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |g_1, \ldots, s_j, \ldots, g_N\rangle$$

Very long lifetime $1/\gamma_{sg} \gtrsim 1$ ms

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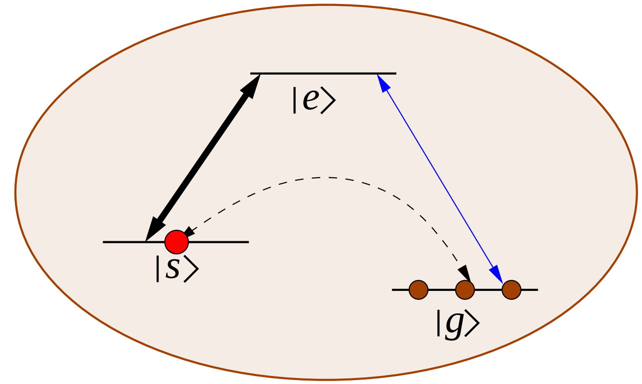

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- $|s^{(1)}\rangle$ symmetric single excitation (spin-flip) state
  equivalent to EIT stored single photon

$\Rightarrow$ **Interconnect SE Qubit and SPh Qubit via EIT**

Fleischhauer, Imamoglu, Marangos, RMP **77**, 633 (2005)

Storage (Ensemble) Qubits

- **Cold Trapped Atoms**:
  - ☹ $N = \rho_a V_a \sim 10^6$ atoms
    $\Rightarrow \sqrt{N} = 10^3$
  - ☹ $|g\rangle = |F=1, M=-1\rangle \leftrightarrow |s\rangle = |F=2, M=1\rangle$ hyperfine transition
    $\Rightarrow$ weak magnetic dipole $\wp_{gs} \sim \mu_{\rm B}/c$
  - ☺ No inhomogeneous broadening $\omega_{sg}^{(j)} = \omega_0$
    $\Rightarrow |s^{(1)}(t)\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\omega_0 t} |g_1, \ldots, s_j, \ldots, g_N\rangle = e^{-i\omega_0 t} |s^{(1)}(0)\rangle$

Verdu et al., PRL **103**, 043603 (2009); Petrosyan et al., PRA **79**, 040304(R) (2009)
Rabl et al., PRL **97**, 033003 (2006); Imamoglu, PRL **102**, 083602 (2009)

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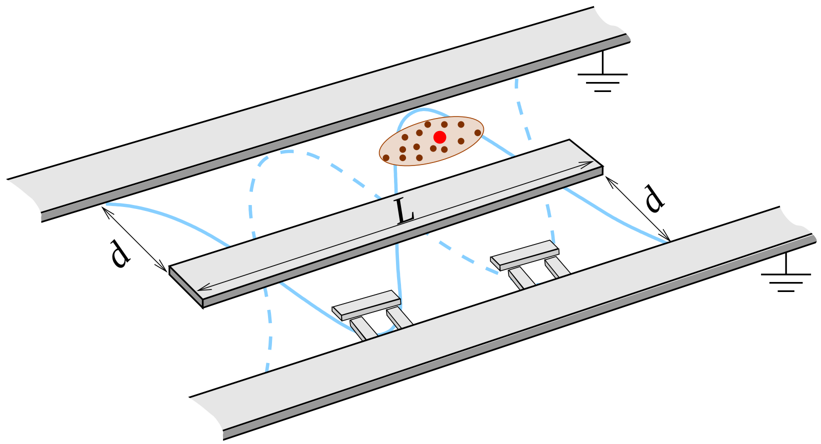
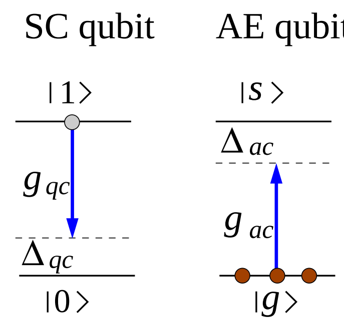
- **Spins (dopants, impurities) in solid**:
  - ☺ $N = \rho_a V_a \sim 10^{12}$ spins
    $\Rightarrow \sqrt{N} = 10^6$
  - ☹ $|g\rangle \leftrightarrow |s\rangle$ magnetic dipole transition $\wp_{gs} \sim \mu_{\rm B}/c$
  - ☹ Large inhomogeneous broadening $\Delta\omega_{sg}^{(j)} \sim 2\pi \times 5 - 10$ MHz
    $\Rightarrow |\tilde{s}^{(1)}(t)\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{-i\omega_j t} |g_1, \ldots, s_j, \ldots, g_N\rangle$

Schuster et al., PRL **105**, 140501 (2010); Kubo et al., PRL **105**, 140502 (2010)
Amsüss et al., PRL **107**, 060502 (2011); Kubo et al., PRL **107**, 220501 (2011)

State Transfer between SC & AE Qubits





$N = \rho_a V_a \sim 10^6$ atoms at $\rho_a \sim 4 \times 10^{13}$ cm^{-3} & $V_a \sim d/2 \times d/2 \times \lambda_c/10$

$|g\rangle \leftrightarrow |s\rangle$ – hyperfine transition ($\omega_{sg}/2\pi = 6.83$ GHz for ^{87}Rb)

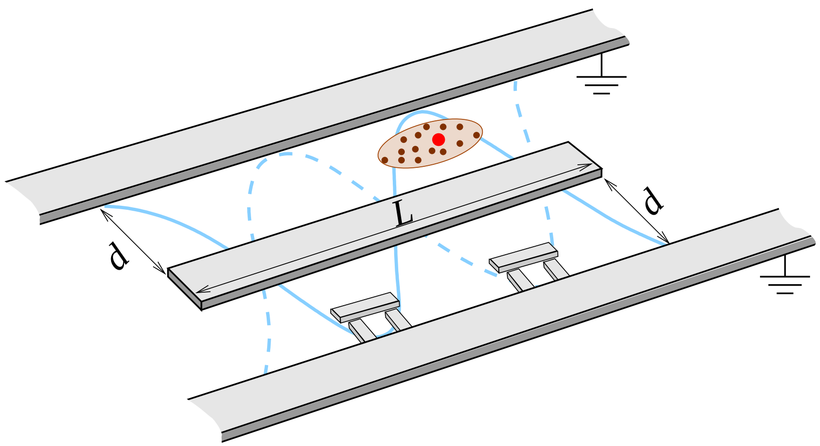
$g_{ac} = i \frac{\wp_{gs}}{\hbar} \varepsilon_c u(\mathbf{r}) \sim 2\pi \times 20$ Hz (magnetic dipole $\wp_{gs} \sim \mu_{\mathrm{B}}/c = \frac{1}{2}\alpha a_0 e$)

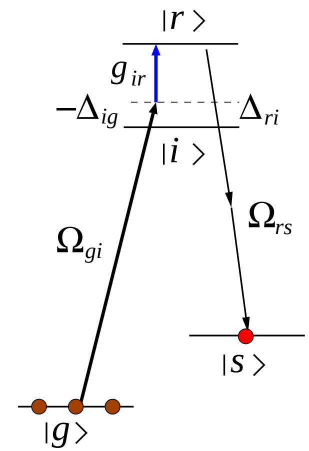
$\Rightarrow \sqrt{N} g_{ac} \lesssim \kappa$ and $\sqrt{N} g_{ac} g_{qc}/\Delta_{qc} \lesssim \gamma_q$

Large Decay & Decoherence

Verdu et al., PRL **103**, 043603 (2009); Imamoglu, PRL **102**, 083602 (2009)

State Transfer between SC & AE Qubits





$|r\rangle$, $|i\rangle$ – Rydberg states with $n \simeq 68$ $\omega_{ri} \simeq \omega_c \simeq 2\pi \times 12$ GHz

$g_{ir} = \frac{\wp_{ir}}{\hbar}\varepsilon_c u(\mathbf{r}) \simeq 2\pi \times 4$ MHz atom–cavity coupling rate (el.-dip. $\wp_{ir} \sim n^2 a_0 e \simeq 10^3 a_0 e$)

$$\boxed{\Delta_{ri} = -\Delta_{ig} \gg g_{ir}, \sqrt{N}\Omega_{gi}} \quad g_{ir} \simeq \sqrt{N}\Omega_{gi} = \Delta/10$$

$\Rightarrow$ Two-photon Rabi frequency $\Omega_{gr}^{(2)} = \frac{\sqrt{N}\Omega_{gi} g_{ir}}{\Delta_{ri}} \sim 0.1 g_{ir}$

Petrosyan et al., PRA **79**, 040304(R) (2009)

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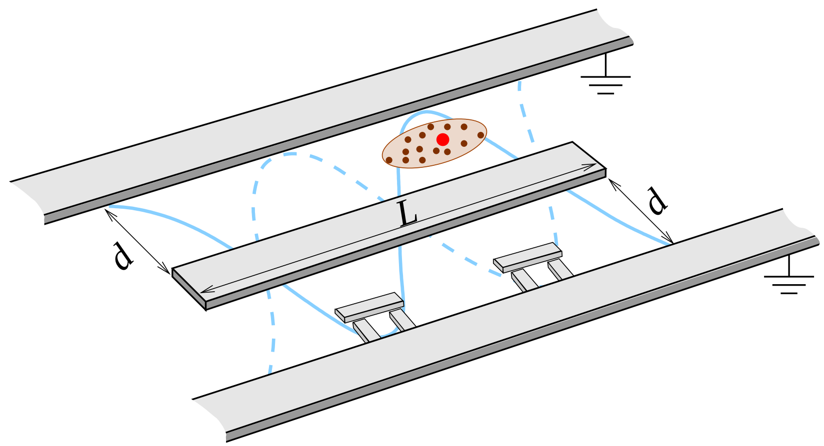

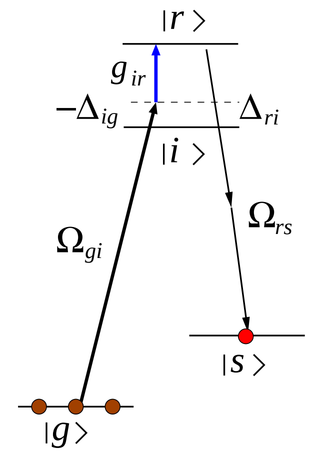

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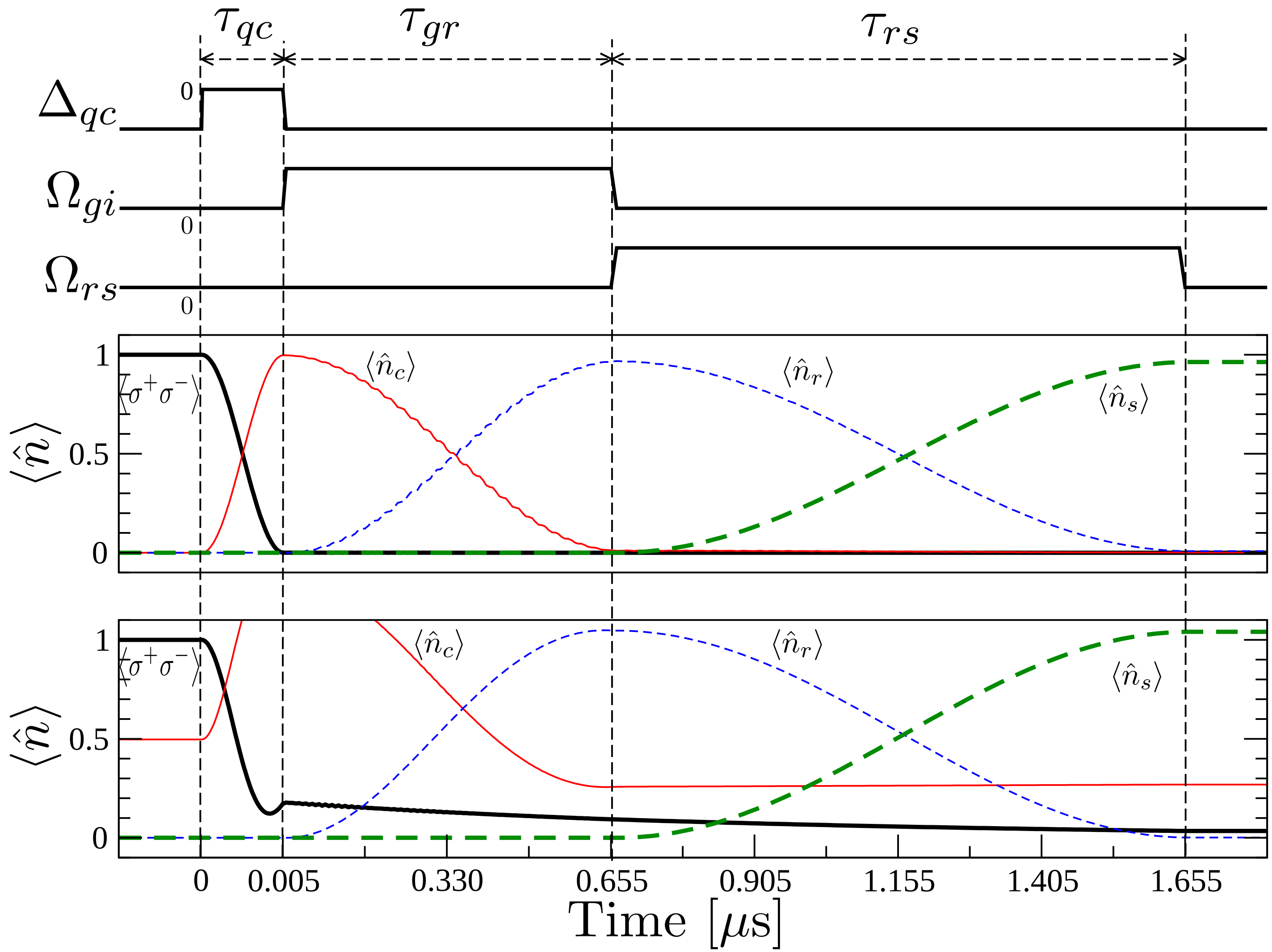
Excitation Transfer from Cavity to Atoms

- Apply Ω_{gi} (π-pulse: $\Omega_{gr}^{(2)} \tau_{gr} = \frac{\pi}{2}$) $\Rightarrow$ $\boxed{|s^{(0)}\rangle\,|1_c\rangle \to |r^{(1)}\rangle\,|0_c\rangle}$
- Apply Ω_{rs} (π-pulse: $\Omega_{rs} \tau_{rs} = \frac{\pi}{2}$) $\Rightarrow$ $\boxed{|r^{(1)}\rangle \to |s^{(1)}\rangle}$

Numerical Simulations

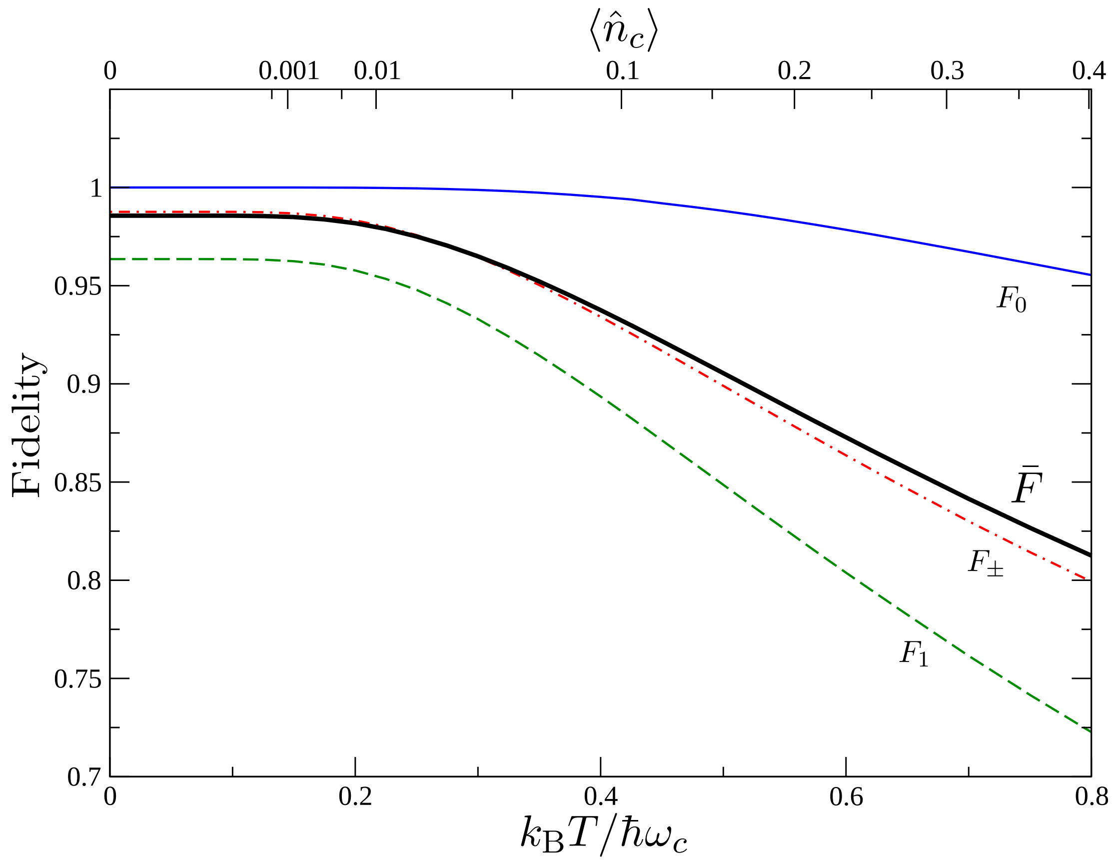


$T = 0$
$[\langle\hat{n}_c\rangle = 0]$

$\frac{k_B T}{\hbar\omega_c} = 0.9$
$[\langle\hat{n}_c\rangle = 0.5]$

$$|\Psi(0)\rangle = |1_q\rangle\,|s^{(0)}\rangle \quad \& \quad \hat{\rho}_c = \frac{e^{-\hbar\omega\hat{n}_c/k_B T}}{\mathrm{Tr}(e^{-\hbar\omega\hat{n}_c/k_B T})}$$

Numerical Simulations



$$F_\psi = \mathrm{Tr}(\hat{\rho}\,|\psi\rangle\langle\psi|\,)$$

$$\bar{F} = \frac{\int F_\psi \, d\,|\psi\rangle}{\int d\,|\psi\rangle}$$

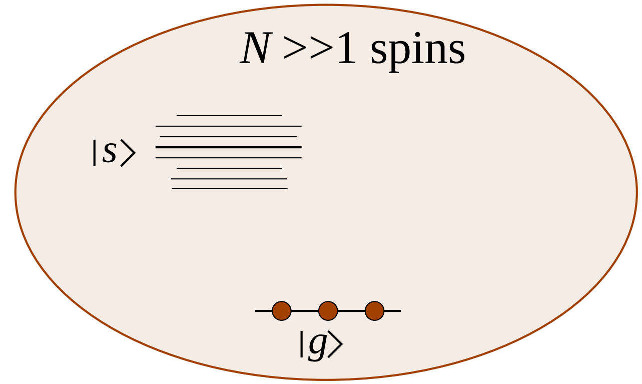
$\bar{F} > 98\%$ for $\frac{k_B T}{\hbar\omega_c} \leq 0.2$ $[\langle \hat{n}_c \rangle = 0.01]$ $\Rightarrow$ $T_{\rm crit} = 0.1$K

Petrosyan et al., PRA **79**, 040304(R) (2009)

Inhomogeneously Broadened SE

$N = \int n(\omega) d\omega$ spins (NV's in diamond)

$n(\omega)$ spectral density of transition $|g\rangle \leftrightarrow |s\rangle$



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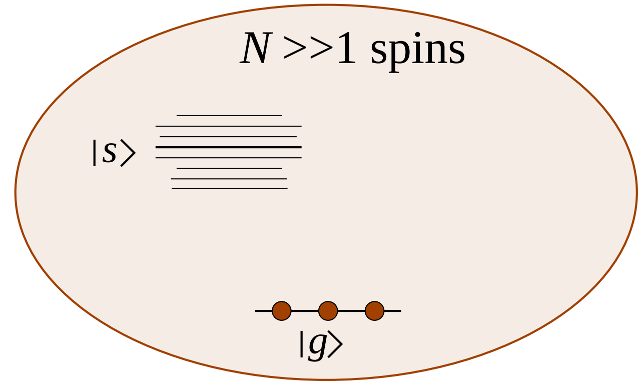

- **Storage fidelity** $|s^{(1)}(0)\rangle \rightarrow |\tilde{s}^{(1)}(t)\rangle$

$$F(t) \equiv |\langle s^{(1)}(t)|\tilde{s}^{(1)}(t)\rangle|^2 = \left| \frac{1}{N} \int n(\omega) e^{i(\omega_0 - \omega_j)t} d\omega \right|^2$$

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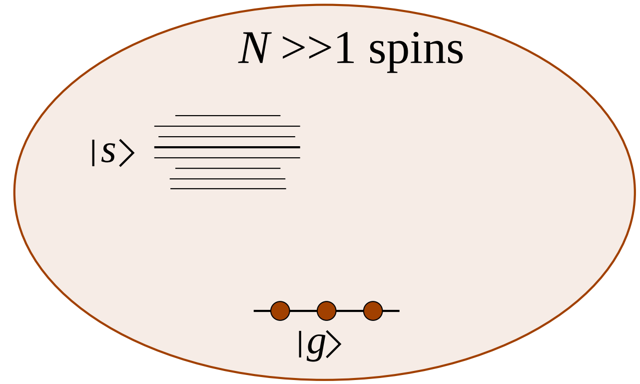

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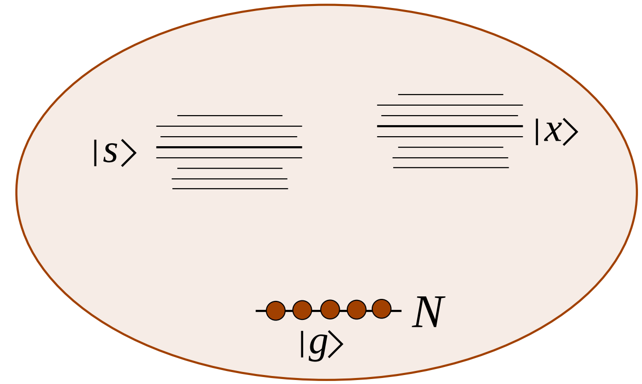
- **QI transfer** $|\Psi(t)\rangle = \alpha(t)\,|1_c, s^{(0)}\rangle + \sum_j \beta_j(t)\,|0_c, s_j\rangle$

$$\dot{\alpha}(t) = -N\bar{\eta}^2 \int_0^t dt' \alpha(t') \sqrt{F(t-t')} \qquad \bar{\eta}^2 \equiv \frac{1}{N} \sum_i |\eta_i|^2$$

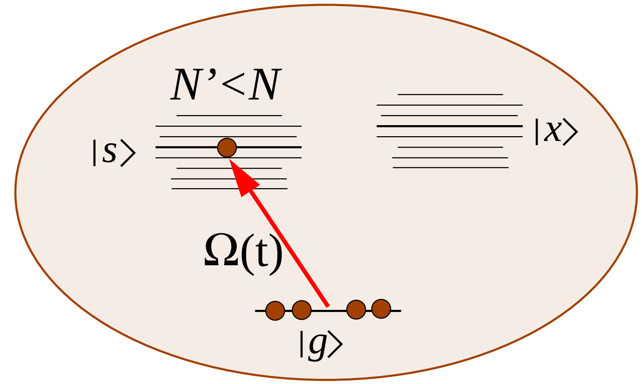
at $t_{\rm tr} = \frac{\pi}{\sqrt{N\bar{\eta}^2}}$ (2π-pulse) $\Rightarrow$ $|\alpha(t_{\rm tr})|^2 \simeq F(t_{\rm tr})$

Filtering SE

$N \sim 10^{12}$ NV's in $|g\rangle$



Filtering SE



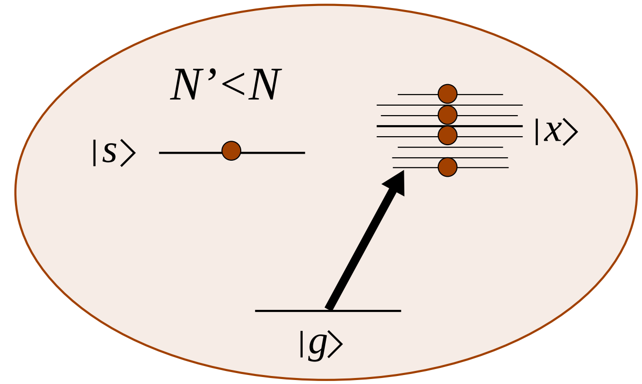
$N \sim 10^{12}$ NV's in $|g\rangle$

(i) Apply $\Omega(t)$ to $|g\rangle \rightarrow |s\rangle$ [$\int_0^T \Omega(t)dt = \frac{\pi}{2}$: π-pulse]

$$\Rightarrow P_s(\omega - \omega_0) = |\langle s| \, e^{-i \int_0^T [(\omega - \omega_0)\sigma_{ss} + \Omega(t)\sigma_{sg}]dt} \, |g\rangle|^2 \quad [P_s(0) = 1]$$

$$N' = \int n(\omega) P_s(\omega - \omega_0) d\omega < N$$

Filtering SE



$N \sim 10^{12}$ NV's in $|g\rangle$

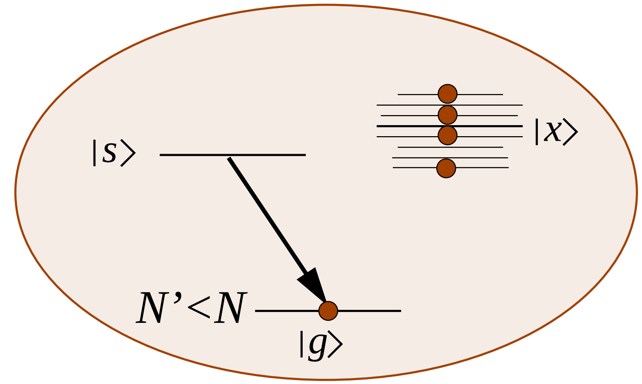
(i) Apply $\Omega(t)$ to $|g\rangle \to |s\rangle$ [$\int_0^T \Omega(t)dt = \frac{\pi}{2}$: π-pulse]

$$\Rightarrow P_s(\omega - \omega_0) = |\langle s| \, e^{-i\int_0^T [(\omega-\omega_0)\sigma_{ss} + \Omega(t)\sigma_{sg}]dt} \, |g\rangle|^2 \quad [P_s(0) = 1]$$

$$N' = \int n(\omega) P_s(\omega - \omega_0) d\omega < N$$

(ii) Transfer all remaining $|g\rangle$ to $|x\rangle$ [adiabat. sweep of $|g\rangle \to |x\rangle$]

Filtering SE



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(ii) Transfer all remaining $|g\rangle$ to $|x\rangle$ [adiabat. sweep of $|g\rangle \to |x\rangle$]

(iii) Return all selected $|s\rangle$ to $|g\rangle$ [adiabat. sweep of $|s\rangle \to |g\rangle$]

Optimal Filtering of SE

Optimal shape of $\Omega(t)$?

$\max[N'\sqrt{F(\tau)}]$ ($\tau < T$) with respect to $\Omega(t)$ $t \in [0, T]$

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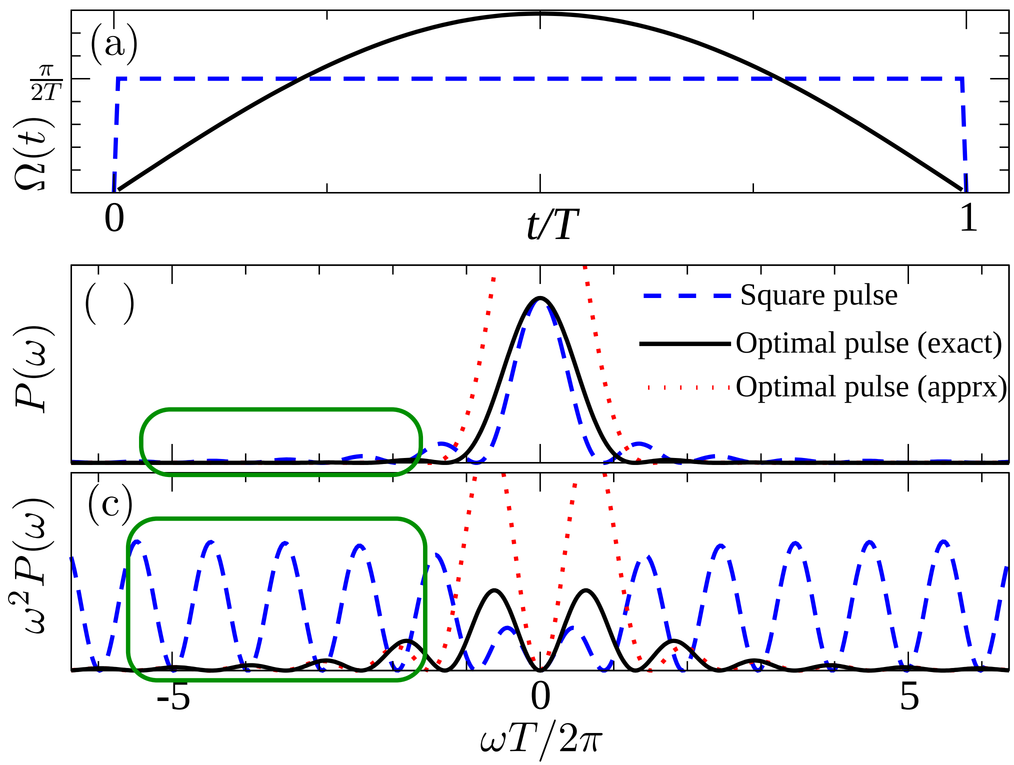
$\max[N'\sqrt{F(\tau)}]$ $(\tau < T)$ with respect to $\Omega(t)$ $t \in [0, T]$

$$\Rightarrow \Omega(t) = \Omega_0 \sin\left(\pi \frac{\lfloor t/\tau \rfloor + 1}{T/\tau + 1}\right)$$

$$\tau \ll T \quad \boxed{\Omega(t) \to \frac{\pi^2}{4T} \sin(\pi t/T)}$$

$$N' \simeq n(\omega_0) \frac{\pi^5}{16T}$$



Bensky et al., PRA (2012); arXiv:1203.6305

Fidelities

☹ **Original SE** (Lorentz. Spct.) $n(\omega) \simeq \frac{n(\omega_0)}{1+(\omega/\Delta)^2} \quad n(\omega_0) = N/\pi\Delta$

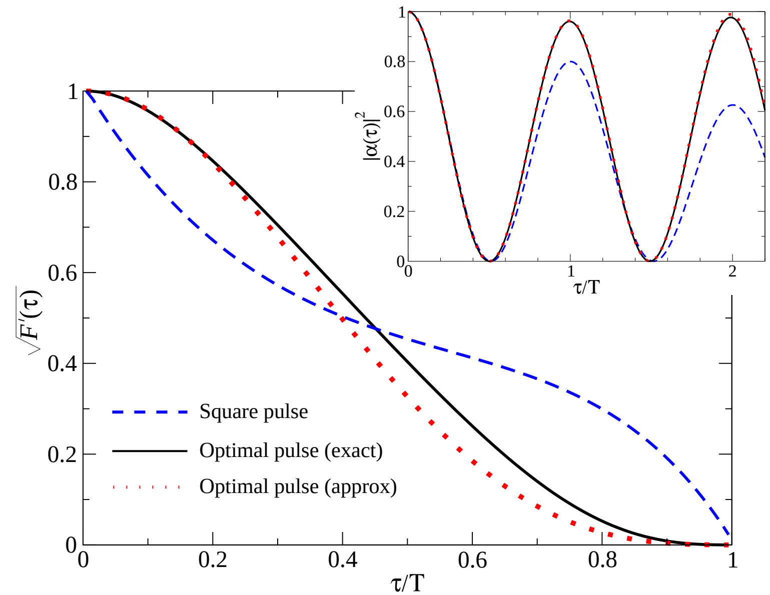
$\Rightarrow \; F(\tau) = e^{-2\Delta\tau} \simeq 1 - 2\Delta\tau$

☹ **Filtered SE** (Square Ω) $N' \simeq n(\omega_0)\frac{3\pi^3}{4T}$

$\Rightarrow \; F(\tau) \simeq 1 - \frac{2}{T}\tau$

☺ **Filtered SE** (Optim. Ω) $N' \simeq n(\omega_0)\frac{\pi^5}{16T}$

$\Rightarrow \; F(\tau) \simeq 1 - \frac{\pi^2}{T^2}\tau^2$



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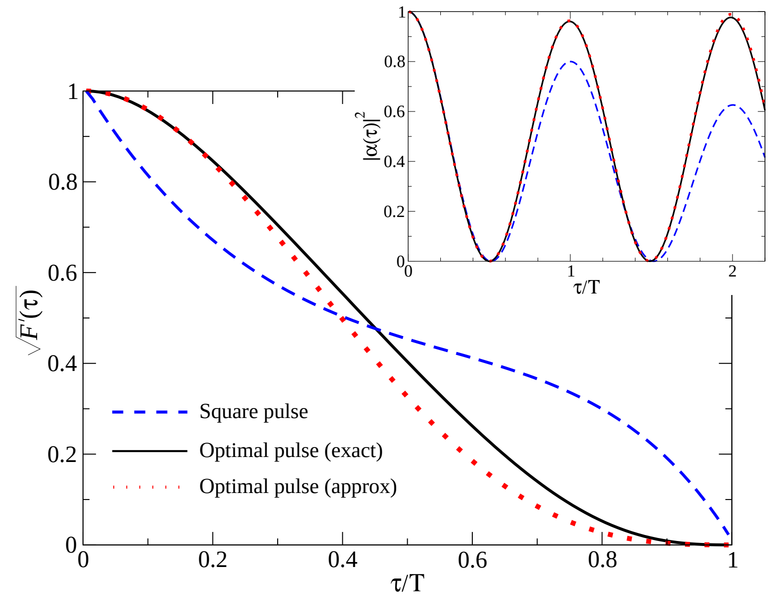


Example [Amsüss et al., PRL **107**, 060502 (2011)]

$N = 10^{12}$ NV's with $\omega_{sg} \simeq 2\pi \times 2.88$ GHz, $\Delta \simeq 2\pi \times 7$ MHz, $\eta_{\text{coll}} = \sqrt{N\bar{\eta}^2} = 2\pi \times 13$ MHz

$\Rightarrow F(\tau) \simeq 1 - \frac{\tau}{60 \text{ ns}} \qquad t_{\text{tr}} \simeq 40 \text{ ns}$

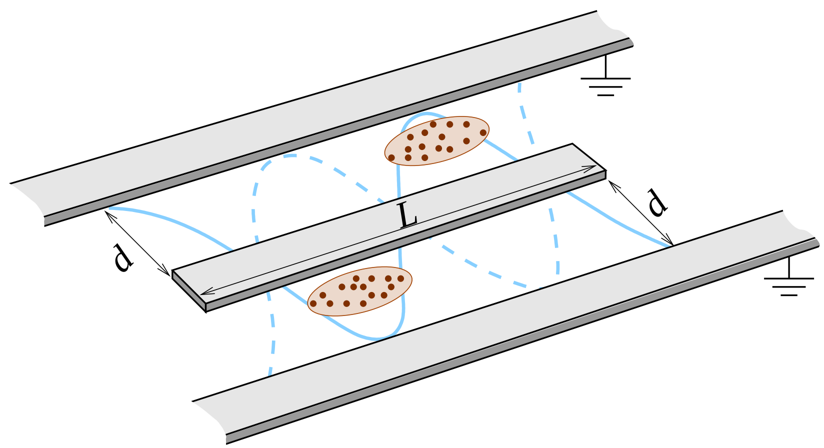
Opt. Filtered SE: $\Omega(t) \to \frac{\pi^2}{4T}\sin(\pi t/T) \quad T = 0.7 \text{ ms}$

$\Rightarrow F(\tau) \simeq 1 - \left(\frac{\tau}{0.22 \text{ ms}}\right)^2 \quad t_{\text{tr}} \simeq 2.8\,\mu\text{s} \ll \kappa^{-1}$

Atom-Atom Interactions in CPW Resonator

Atoms in CPW Resonator

Ensembles of cold ground-state atoms trapped near field antinodes

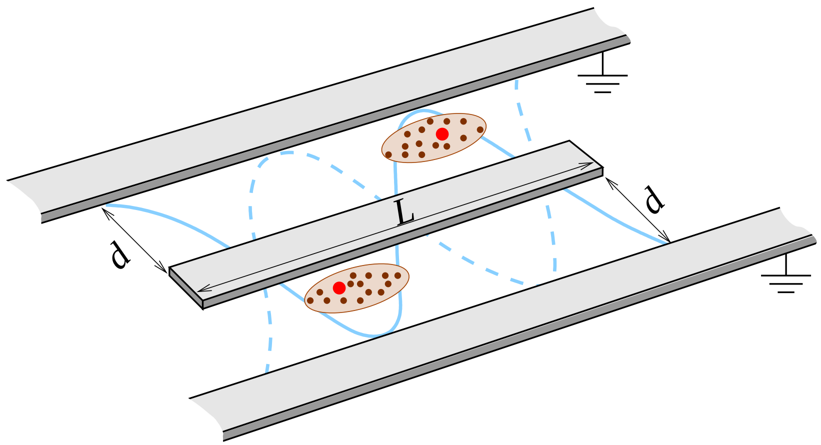


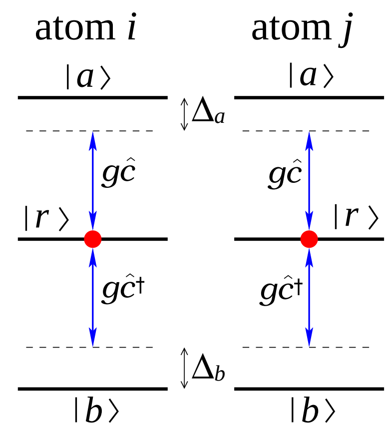
$N = \rho_a V_a \sim 10^6$ atoms in each ensemble

Petrosyan, Fleischhauer, PRL **100**, 170501 (2008)

Atoms in CPW Resonator

Excited Rydberg atoms i & j interact via common cavity mode





Hamiltonian for atom i + atom j + cavity field:

$$H = \hbar \sum_{l=i,j} [(\Delta_a \hat{\sigma}^l_{aa} - \Delta_b \hat{\sigma}^l_{bb}) + (g^l_{br} \hat{c}^\dagger \hat{\sigma}^l_{br} + g^l_{ar} \hat{c}\, \hat{\sigma}^l_{ar} + \mathrm{H.c.})]$$

$\Delta_a = \omega_{ar} - \omega_c$ and $\Delta_b = \omega_{rb} - \omega_c$ detunings

$\hat{\sigma}^l_{\mu\nu} = |\mu_l\rangle\langle\nu_l|$ atomic transition operator $\hat{c}$, $\hat{c}^\dagger$ cavity field operators

$g^l_{\mu\nu} = -\frac{\wp_{\mu\nu}}{\hbar} \varepsilon_c u(\mathbf{r}_l)$ atom-field coupling ($g^l_{br} \approx g^l_{ar} \equiv g_r \sim 2\pi \times 10$ MHz)

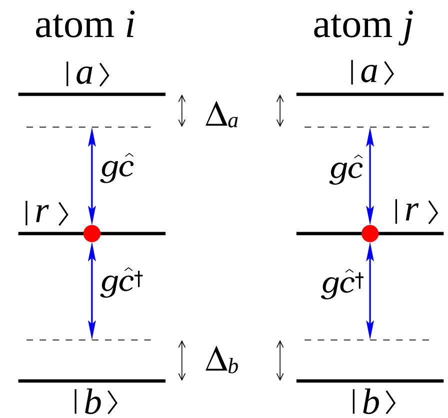
Petrosyan, Fleischhauer, PRL **100**, 170501 (2008)

Resonant Dipole-Dipole Interaction

$$\Delta_{a,b} \simeq g_r f \gg g_r \quad (f \gg 1)$$

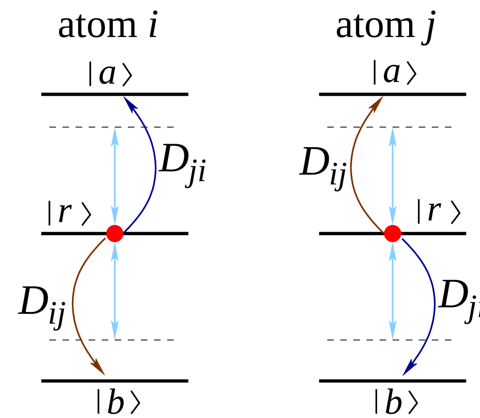
- $|r_i\rangle\,|r_j\rangle\,|0_c\rangle \to |r_{i,j}\rangle\,|b_{j,i}\rangle\,|1_c\rangle$ (1Ph) nonresonant
- $|r_i\rangle\,|r_j\rangle\,|0_c\rangle \to |a_{i,j}\rangle\,|b_{j,i}\rangle\,|0_c\rangle$ (2Ph) resonant

  (ac Stark compensated: $\Delta_a - \Delta_b + s_a^{i,j} - s_r^i - s_r^j = 0$)



$\Downarrow$

Resonant Dipole-Dipole Interaction



Effective cavity-mediated RDDI between atoms i and j

Effective Hamiltonian (2nd-order in g_r):

$$V_{ij}^{(2)} = \hbar D_{ij} \left(\hat{\sigma}_{br}^{i} \hat{\sigma}_{ar}^{j} + \hat{\sigma}_{ar}^{i} \hat{\sigma}_{br}^{j} \right) + \mathrm{H.c}$$

$D_{ij} = \frac{g_{br}^{i} g_{ar}^{j}}{\Delta_b} = \frac{g_{ar}^{i} g_{br}^{j}}{\Delta_b} = \frac{g_r}{f} \equiv D$ — RDDI constant

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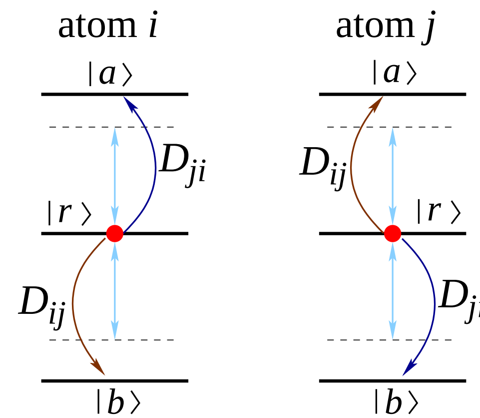

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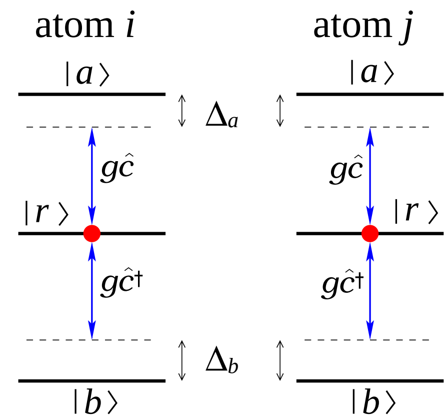
Eigenstates of $V_{ij}^{(2)}$

$$|\psi_{ij}^{0}\rangle = \frac{1}{\sqrt{2}} \left(|b_i\rangle |a_j\rangle - |a_i\rangle |b_j\rangle \right) \qquad \lambda_0 = 0$$

$$|\psi_{ij}^{\pm}\rangle = \frac{1}{\sqrt{2}} |r_i\rangle |r_j\rangle \pm \frac{1}{2} \left(|b_i\rangle |a_j\rangle + |a_i\rangle |b_j\rangle \right) \qquad \lambda_{\pm} = \pm \hbar \sqrt{2} D_{ij}$$

Van der Waals Interaction

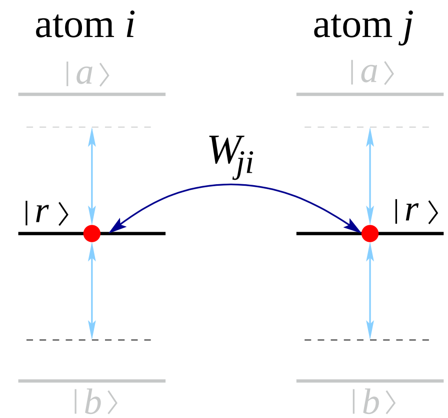
$$\boxed{\Delta_a \simeq g_r(f-1) \quad \& \quad \Delta_b \simeq g_r f \qquad (f \gg 1)}$$



- 1ph and 2ph transitions nonresonant

$$\Downarrow$$

Van der Waals Interaction



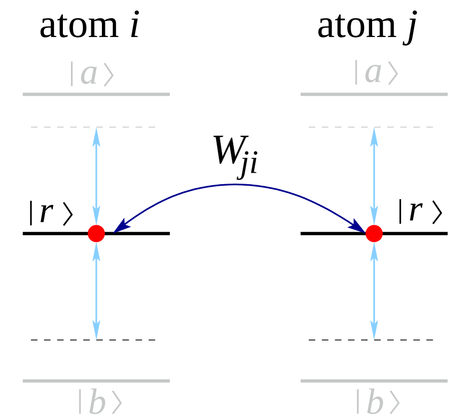
Effective cavity-mediated VdWI between atoms i and j

Effective Hamiltonian (4th-order in g_r):

$$V_{ij}^{(4)} = \hbar\, \hat{\sigma}_{rr}^{i} W_{ij} \hat{\sigma}_{rr}^{j}$$

$W_{ij} = \frac{2|g_{br}^{i} g_{br}^{j}|^2}{\Delta_b^3} - \frac{2|g_{br}^{i} g_{ar}^{j}|^2}{(\Delta_a - \Delta_b)\Delta_b^2} = \frac{4 g_r}{f^3} \equiv W$ — WdVI constant

Van der Waals Interaction



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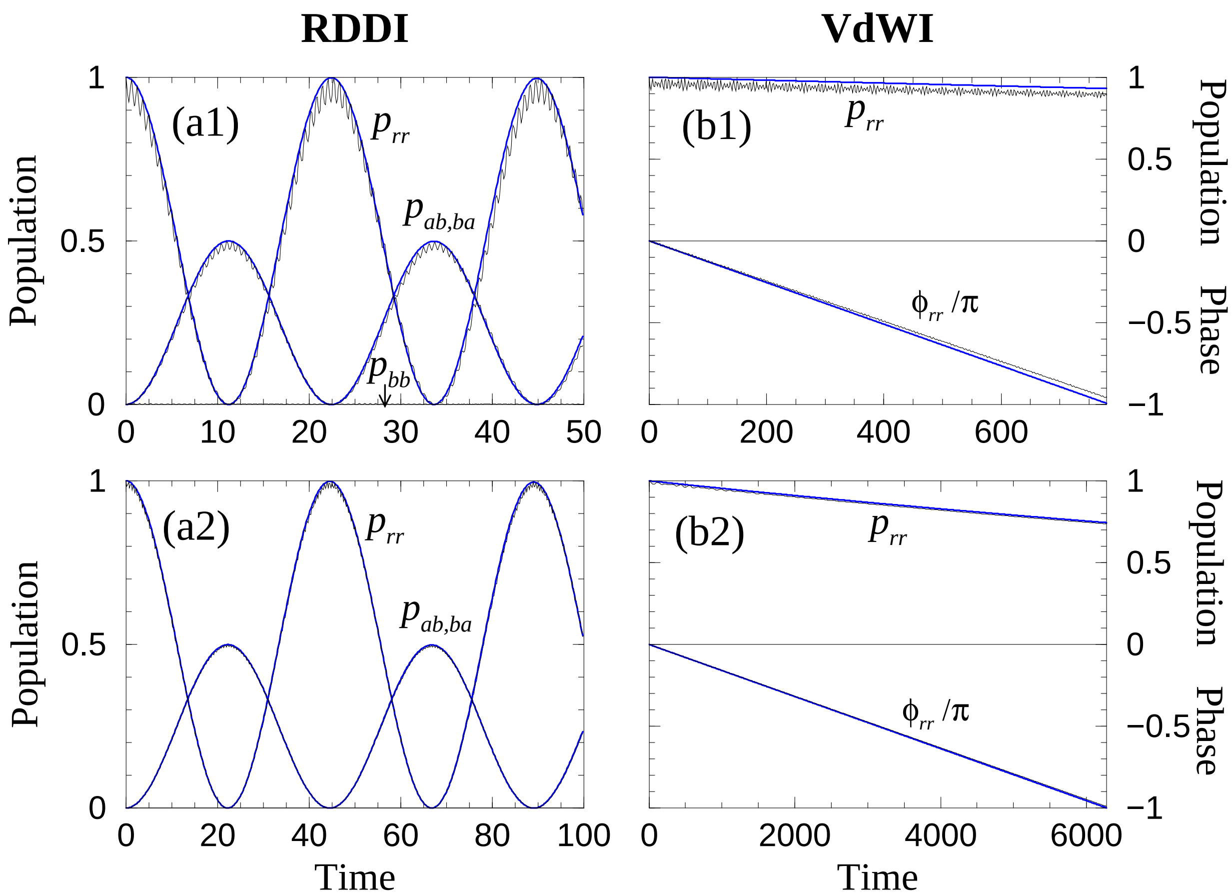
Evolution due to $V_{ij}^{(4)}$

$$|r_i\rangle\,|r_j\rangle \to e^{i\phi_{rr}(t)}\,|r_i\rangle\,|r_j\rangle \quad \phi_{rr}(t) = Wt$$

Numerical Simulations

Dynamics of Rydberg atoms i , j in CPW cavity, $|\Phi(0)\rangle = |r_i\rangle |r_j\rangle |0_c\rangle$



(a1),(b1) $f = 10$

(a2),(b2) $f = 20$

$p_{\mu\nu}$ populations of $|\mu_i\rangle |\nu_j\rangle$ ϕ_{rr} phase-shift of $|r_i\rangle |r_j\rangle$

"——": ME simulations for the full system using Hamiltonian H

"——": ME simulations for the effective Hamiltonians $V_{ij}^{(2)}$ and $V_{ij}^{(4)}$

Applications of RDDI

Intracavity Dipole Blockade $[D \simeq 2\pi \times 1$ MHz $(f = 10)]$

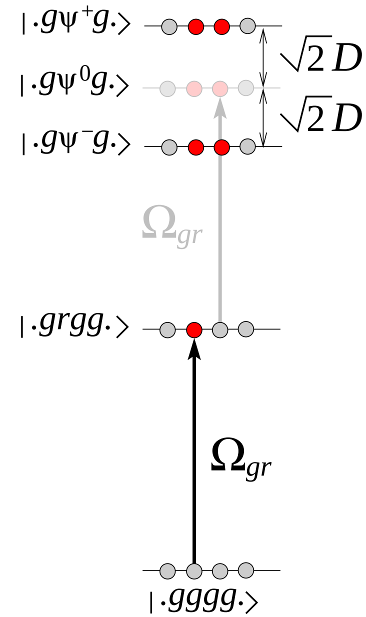
$\langle rg|\,\hat{\sigma}_{gr}\Omega_{gr}\,|\psi^0\rangle = 0$ & $\Omega_{gr} < D \quad \Rightarrow$

Single Rydberg excitation of atomic ensemble by Ω_{gr}

(effective π-pulse: $\sqrt{N}\Omega_{gr}T_1 = \frac{\pi}{2}$)

$$\boxed{|s^{(0)}\rangle \to |r^{(1)}\rangle}$$

$$|s^{(0)}\rangle \equiv |g_1, g_2, \ldots, g_N\rangle \quad |r^{(1)}\rangle \equiv \frac{1}{\sqrt{N}}\sum_i e^{i\mathbf{k}_{gr}\cdot\mathbf{r}_i}\,|g_1, \ldots, r_i, \ldots, g_N\rangle$$



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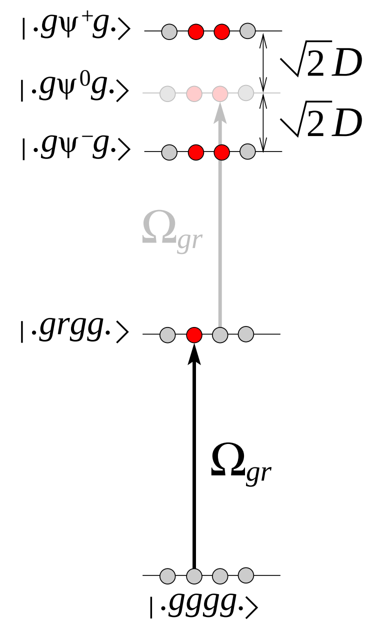


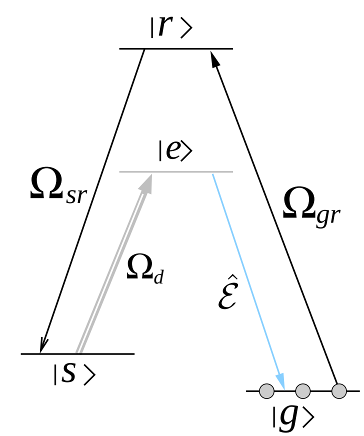
Single Photon Generation

- Apply Ω_{sr} (π-pulse: $\Omega_{sr}T_2 = \frac{\pi}{2}$)

$$|r^{(1)}\rangle \rightarrow |s^{(1)}\rangle$$

$|s^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_i e^{i\delta\mathbf{k}\cdot\mathbf{r}_i} |g_1, \ldots, s_i, \ldots, g_N\rangle$

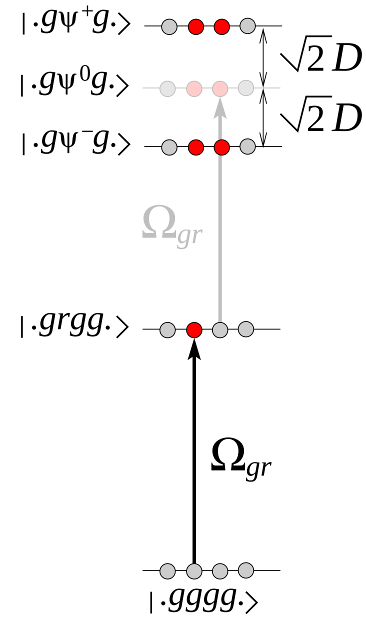
single collective Raman excitation of atomic ensemble
(equivalent to EIT stored single photon)



- Turn on $\Omega_d \rightarrow$ release single photon pulse $\hat{\mathcal{E}}$

Applications of RDDI

Intracavity Dipole Blockade $[D \simeq 2\pi \times 1$ MHz $(f = 10)]$



$\langle rg| \hat{\sigma}_{gr}\Omega_{gr} |\psi^0\rangle = 0$ & $\Omega_{gr} < D \quad \Rightarrow$

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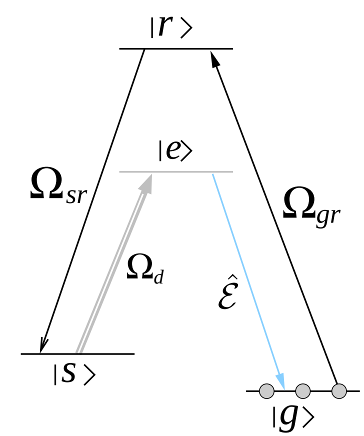
$$\boxed{|s^{(0)}\rangle \to |r^{(1)}\rangle}$$

$$|s^{(0)}\rangle \equiv |g_1, g_2, \dots, g_N\rangle \quad |r^{(1)}\rangle \equiv \frac{1}{\sqrt{N}} \sum_i e^{i\mathbf{k}_{gr}\cdot\mathbf{r}_i} |g_1, \dots, r_i, \dots, g_N\rangle$$

Entangling two (or more) ensembles A and B

- Apply Ω_{gr} simultaneously to A and B ($\sqrt{2N}\Omega_{gr}T_1 = \frac{\pi}{2}$)
- Apply Ω_{sr} (π-pulse: $\Omega_{sr}T_2 = \frac{\pi}{2}$)

$$\Rightarrow \frac{1}{\sqrt{2}}\left(|s^{(1)}\rangle_A |s^{(0)}\rangle_B + |s^{(0)}\rangle_A |s^{(1)}\rangle_B \right)$$



Applications of VdWI

Dispersive phase shift $[W \simeq 2\pi \times 40$ KHz $(f = 10)]$

$$\boxed{|r\rangle\,|r\rangle \rightarrow e^{i\phi(t)}\,|r\rangle\,|r\rangle \quad \phi(t) = Wt}$$

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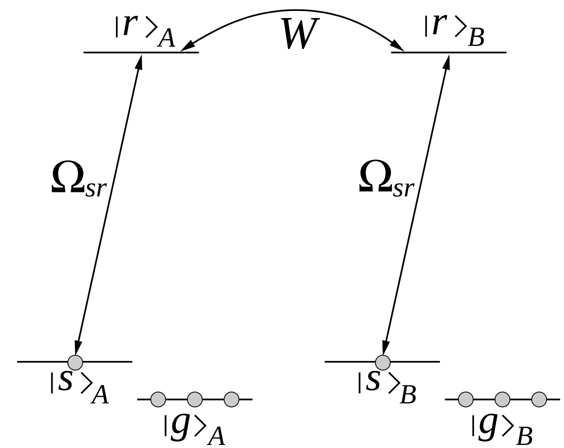
$$|r\rangle|r\rangle \to e^{i\phi(t)}|r\rangle|r\rangle \quad \phi(t) = Wt$$

Ensemble qubits $A, B, \ldots$ in states

$$|\psi\rangle_q = \alpha_q |s^{(0)}\rangle_q + \beta_q |s^{(1)}\rangle_q$$



- Apply Ω_{sr} to A & B ($\Omega_{sr}T_1 = \frac{\pi}{2}$):

$$|s^{(1)}\rangle_{A,B} \to |r^{(1)}\rangle_{A,B}$$

- Phase-shift $\phi = WT_\pi = \pi$ (during $T_\pi = \pi/W$)
- Apply Ω_{sr} again ($\Omega_{sr}T_2 = \frac{\pi}{2}$):

$$|r^{(1)}\rangle_{A,B} \to |s^{(1)}\rangle_{A,B}$$

$\Rightarrow$ Universal CPHASE gate between ensemble qubits A & B

$$|s^{(x)}\rangle_A |s^{(y)}\rangle_B \to (-1)^{xy} |s^{(x)}\rangle_A |s^{(y)}\rangle_B \qquad (x, y \in [0, 1])$$

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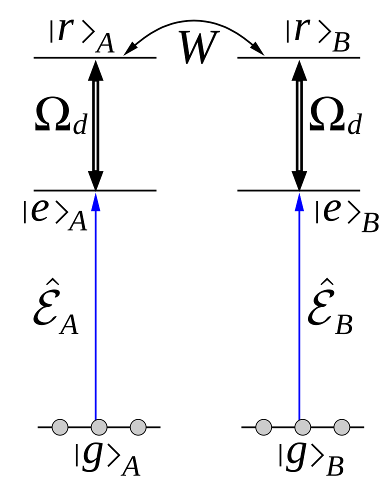
Photonic qubits $A, B, \ldots$ in states
$|\psi\rangle_q = \alpha_q\,|0\rangle_q + \beta_q\,|1\rangle_q$



- EIT dark-state polaritons in ensembles A & B

$$\Psi_{A,B} = \cos\theta\,\hat{\mathcal{E}}_{A,B} - \sin\theta\,\sqrt{N}\hat{\sigma}_{gr}^{A,B}$$

- Phase shift $\phi = WT_{\text{int}} = \pi \quad (T_{\text{int}} = \frac{L_a}{v_g}, v_g = c\cos^2\theta \ll c)$

$\Rightarrow$ Universal CPHASE gate between photonic qubits A & B
$|x\rangle_A\,|y\rangle_B \to (-1)^{xy}\,|x\rangle_A\,|y\rangle_B \qquad (x, y \in [0, 1])$

Conclusions: Efficient QI Processing

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  - solid state SC qubits – fast quantum gates
  - metastable states of atoms (dopants) – reliable quantum memory
  - single photons – long-distance information transfer

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  - EIT provides interface between static (AE) and flying (SPh) qubits
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    - *serve as quantum bus for SC and AE qubits*
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- **Experimental efforts**
  - CPW, SC qubits & SEs on atom chips
  - Free-space Rydberg blockade via DDI [r_{ij}^{-3}] and VdWI [r_{ij}^{-6}]
  - EIT in warm & cold AEs, OLs [$1/\gamma_{sg} \gtrsim 1$ s], doped solids ...

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TU Wien

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EMALI

MIDAS

Macroscopic Interference Devices
for Atomic and Solid-State Systems