# Quantum-nondemolition measurement of the number of photons in a microcavity

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(Submitted 23 May 1997)
Pis'ma Zh. Éksp. Teor. Fiz. **66**, No. 1, 58–63 (10 July 1997)

A new scheme for quantum-nondemolition measurement of the number and statistics of photons in a microwave cavity on the basis of interferometry of strongly-controlled $V$ atoms which interact by a dispersion interaction with a cavity mode at a different transition is proposed. © *1997 American Institute of Physics.* [S1063-7761(97)01313-9]

PACS numbers: 42.50.Ar, 42.55.Sq

**1**. This letter proposes a new scheme for quantum-nondemolition measurement of the number of photons in a microcavity. The scheme is based on interferometry of $V$ atoms which interact by a dispersion interaction with a cavity mode at the transition $1\rightarrow 3$ (Fig. 1) and are controlled by a coherent field at a neighboring transition $1\rightarrow 2$. We show that inside the cavity the dressed states of the system ''atom + coherent field'' undergo different phase shifts which are induced by the quantum field and therefore depend on the number of photons $n$ in the cavity. When the appropriate conditions are satisfied, $n$ remains unchanged over the transit time of the atoms through the cavity. At the same time, the phase shifts lead to observable effects in the distribution of atoms which leave the cavity in the states 1 and 2; this makes it possible to measure the number of photons of the cavity field without changing its value. The dependence of the populations of the atomic levels on the phase shifts arises on account of the nonadiabatic switching on and off of the control field at the entrance and exit of the cavity; this results in a strong mixing of the dressed states or Ramsey interference. Nondemolition measurement of a small number of photons in a cavity has been discussed in Refs. 1–3. In Refs. 1 and 2, a Ramsey interferometry scheme in separated oscillatory fields was considered for observing the dispersion phase shift of nonresonant Rydberg atoms. In our case, throughout its entire flight through the cavity an atom interacts simultaneously with both a cavity mode and the Ramsey field; this is close to the experimental situation in microcavities.[4] We note that Ramsey-type interference, which is produced by nonadiabatic mixing of the upper and lower states of a two-level Rydberg atom, was observed in these experiments.

**2**. Let us consider the interaction of a $V$ atom with the field of a single-mode cavity at frequency $\omega_c$ detuned from the frequency of the atomic transition $1\rightarrow 3$ by the amount $\Delta_c=\omega_{31}-\omega_c\neq 0$. The atom is controlled by an external classical field $E$ at the frequency $\omega_1=\omega_{21}$ (Fig. 1a), whose intracavity Rabi frequency equals $\Omega_0=g_0E/\delta_c$ (Refs. 5 and 6), where $g_0$ is the coupling constant between the atom and the electromagnetic field at the transition $1\rightarrow 2$ and $\delta_c=\omega_c-\omega_1$ is the detuning of the field $E$ from the cavity. Cavities

  

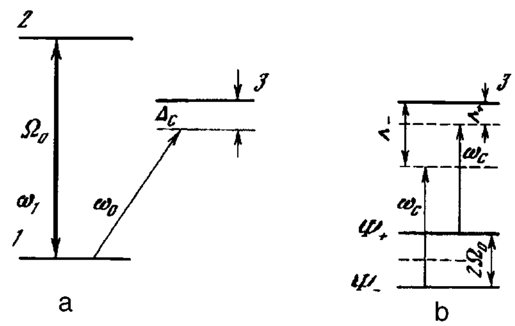


FIG. 1. Configuration of the levels of a $V$ atom in the basis of bare (a) and dressed (b) states of the system. $\Omega_0$ is the Rabi frequency of the control field.

with three openings for entrance and exit of atoms and for injection of an external field were recently used in experiments in Ref. 7.

Let us assume first that the cavity mode is in a Fock state with $n$ photons. It is obvious that the interaction with the atom does not change $n$, provided that both the detunings $\Delta_c$ and $\delta_c$ are so large that as the atom passes through the cavity the photon absorption on both transitions $1\rightarrow 2$ and $1\rightarrow 3$ can be neglected. A pair of dressed states of the system ''atom + classical field'' $|\Psi_\pm\rangle = (1/\sqrt{2})(|1\rangle \pm |2\rangle)$, which are split by an amount equal to the Stark splitting $2\Omega_0$ (Fig. 1b), is formed inside the cavity. In Ref. 6 it was shown that the transition rates of an atom from $|\Psi_\pm\rangle$ into the level 3 with the absorption of one cavity photon are equal to, respectively,

$$\Gamma_\pm = \frac{g_1^2 kn}{k^2 + \Lambda_\pm^2}. \tag{1}$$

Here $g_1$ is the coupling constant between the atom and the electromagnetic field at the transition $1\rightarrow 3$ and $k$ is the rate of decay of the number of photons in the cavity: $k \ll \Omega_0$ and $\Lambda_\pm = \pm \Omega_0 - \Delta_c$. If the transit time of the atom through the cavity equals $T$, then the condition for a dispersion interaction at the transition $1\rightarrow 3$ has the form $\Gamma_\pm T \ll 1$, which in the case of large detuning $\Delta_c \gg \Omega_0 \gg k$ reduces to

$$g_1^2 nkT/\Delta_c^2 \ll 1 \tag{2}$$

or

$$\Delta_c \gg \Delta_{\text{th}} = g_1 (nkT)^{1/2}. \tag{3}$$

The restriction on $\delta_c$ is stronger and follows from the obvious condition $\omega_{23} \gg 2\Omega_0$, which gives $\delta_c = \omega_{23} + \Delta_c \gg \Omega_0$.

Let us now consider the evolution of the atomic wave functions under the conditions described. In the experiment, the atoms are prepared and detected outside the cavity. For this reason, regions where both fields are switched on and off are present at the entrance and exit from the cavity, and therefore the coupling constants $g_0$ and $g_1$ for a monokinetic beam of atoms are functions of time, they are constant inside the cavity and vary continuously in the cavity openings. However, if on account of a large detuning $\Delta_c$ the interaction with the cavity mode with moderate atom velocities is adiabatic everywhere, then the switching on and off of a classical field which is in exact resonance with the atom is always of a nonadiabatic character. Let us represent the Rabi frequency $\Omega(t)$ in the form

$$\Omega(t)=\begin{cases}\Omega_0=g_0E/\delta_c, & 0\leqslant t\leqslant T\\ \Omega_0\times f_1(t), & -\infty<t\leqslant 0,\\ \Omega_0\times f_2(t), & T\leqslant t\leqslant\infty\end{cases} \tag{4}$$

where $f_{1,2}(t)$ are normalized so that $f_1(0)=f_2(T)=1$ and $f_1(-\infty)=f_2(\infty)=0$. Then the atom, initially prepared in state 1 and passing through the region where the control field is switched on, is in a superposition state

$$|\Phi\rangle=a_1(0)|1\rangle+a_2(0)|2\rangle, \tag{5}$$

where

$$a_1(t)=\cos\ \eta_1(t),\quad a_2(t)=-i\ \sin\ \eta_1(t) \tag{6}$$

$$\eta_1(t)=\int_{-\infty}^{t}\Omega(\tau)d\tau=\Omega_0,$$

$t\leqslant 0$, is the area of the the field envelope in this region. The equation (5) describes the Ramsey interference due to nonadiabatic interaction of the atoms with the field $E$. Inside the cavity it is convenient to switch to a basis of dressed states, representing $|\Phi\rangle$ in the form

$$|\Phi\rangle=b_+(0)|\Psi_+\rangle+b_-(0)|\Psi_-\rangle, \tag{7}$$

where

$$b_\pm(t)=\frac{1}{\sqrt{2}}[a_1(t)\pm a_2(t)],\quad 0\leqslant t\leqslant T. \tag{8}$$

Using the Hamiltonian describing the interaction of the atom with the quantum field of the cavity in a basis of dressed states[6] and also Eq. (6) as initial conditions, we easily find the amplitudes $b_\pm(T)$:

$$b_\pm(T)=b_\pm(0)\,\delta_\mp(T)+i\frac{r}{\varphi}\ \exp(irT)\sin(\varphi T)b_\mp(0), \tag{9}$$

where $r=g_1^2n/(2\Delta_c)$, $\varphi=\sqrt{r^2+\Omega_0^2}$, and

$$\delta_\pm(T)=\left[\cos(\varphi T)\pm i\ \frac{r}{\varphi}\sin(\varphi T)\right]\ \exp(irT).$$

We note that in Eq. (9) the cavity-field-induced phase shifts of the states $|\Psi_\pm\rangle$ are taken into account to all orders. It is also easy to verify that

$$|b_+(T)|^2+|b_-(T)|^2=|b_+(0)|^2+|b_-(0)|^2=1,$$

i.e., the level 3 is not excited by absorption of a photon from the cavity field and therefore the number of photons $n$ is conserved.

The amplitudes $a_{1,2}(T)$ of the bare states are obtained by an inverse transformation from Eq. (8). Using them as initial conditions for the Schrödinger equation in the region $t \geqslant T$, we find the solution for these amplitudes at the detector, i.e., in the limit $t \rightarrow \infty$:

$$a_{1,2}(\infty) = a_{1,2}(T)\cos\eta_2(\infty) - i a_{2,1}(T)\sin\eta_2(\infty), \tag{10}$$

where

$$\eta_2(t) = \Omega_0 \int_T^t f_2(\tau)d\tau,$$

$t \leqslant T$, is the area of the envelope of the control field in the region where the field is switched off. The secondary Ramsey interference at the exit from the cavity makes it possible to preserve information about the phase shifts in the populations of the atomic levels $P_i(\infty) = |a_i(\infty)|^2$, $i = 1, 2$, which are measured in the experiment as a function of the number $n$ of photons, the detuning $\Delta_c$, and the transit time $T$ of the atoms. If the cavity field is not in a Fock state, then the probability of counting atoms as a function of $\Delta_c$ is measured in the experiment as an average over the photon distribution $p(n)$:

$$P_i(\Delta_c, t) = \sum_n p(n) P_i(\infty, n, \Delta_c, T). \tag{11}$$

In a real experiment, however, we deal with a thermal beam of Rydberg atoms, and therefore $P_i(\Delta_c, T)$ must also be averaged over the velocities of the atoms or over the transit times $T = L/v$, where $L$ is the cavity length and $v$ is the velocity of the atoms. It must be kept in mind, however, that the dependence of $P_i$ on the photon statistics vanishes under such averaging, if the temporal distribution has a width greater than $r^{-1}$ or if the width of the velocity distribution of the atoms is greater than $\Delta v = v_0^2 (Lr)^{-1}$, where $v_0$ is the most probable velocity of the atoms. We shall estimate $\Delta v$ on the basis of the following considerations. It is known that random electromagnetic fields localized in cavity openings result in uncontrollable Ramsey interference for incoming and outgoing atoms.[4] This effect is absent for slow atoms, whose interaction with random fields can be assumed to be adiabatic. Under the conditions of the experiment performed in Ref. 4, the time of flight of these atoms is of the order of $8 \times 10^{-5}$ s, which with $L \cong 2.5$ cm corresponds to velocities $v_0 \cong 300$ m/s. In further estimates, for reliability, we take $T \geqslant 4 \times 10^{-4}$ s. Since the coupling constant $g_1$ for Rydberg atoms is ordinarily equal to $10^5$ s$^{-1}$, for $n \cong 10$, $\Delta_c \cong 200$ kHz, and correspondingly $r \cong 2 \times 10^5$ s$^{-1}$, we find that the ratio $\Delta v / v_0$ equals several percent (see also Ref. 8). Then velocity averaging has virtually no effect on the final results. Therefore, fixing $v_0$, we can choose $\Omega_0$ so that $\eta_1(0) = \eta_2(\infty) = \pi/2$. As a result, we obtain simple expressions for the populations of the atoms. Specifically, in the case of a cavity mode in a Fock state we have for $P_i$

$$P_2 = \frac{\Omega_0^2}{\varphi^2} \sin^2\left(\varphi \frac{L}{\nu_0}\right), \quad P_1 = 1 - P_2. \tag{12}$$

For $\omega_c \cong 20-50$ GHz and cavity $Q \cong 10^9$, which corresponds to damping $k \cong 10-25$ Hz, we find from Eq. (3) with $n = 10$ that $\Delta_{\rm th} \cong 20-30$ kHz. In Fig. 2 we display $P_2$ (11) and

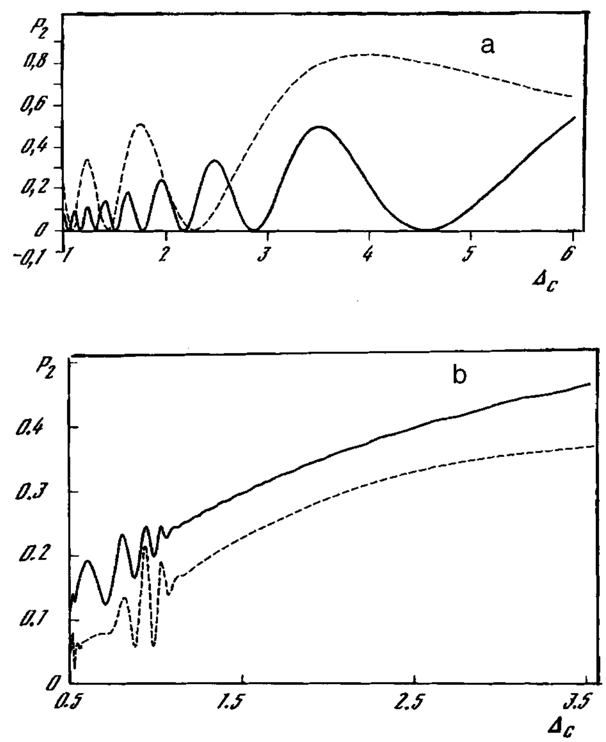


FIG. 2. Population of the upper level 2 of a monokinetic atomic beam at the exit from the cavity as a function of the detuning $\Delta_c$ (in units of $4\Omega_0$, $\Omega_0 = 50$ kHz): a — Cavity field in a Fock state: $n = 10$ (dashed curve) and $n = 25$ (solid curve); b — coherent field (dashed curve) and thermal field (solid curve). The average number of photons in the last two cases is $\langle n \rangle = 10$.

(12) versus the detuning $\Delta_c$ for $\Delta_c \geqslant 100$ Hz and three states of the cavity field. One can see that in all cases the oscillations in $P_2$ vanish with increasing $\Delta_c$ in the region where $r < \Omega_0$, i.e., when the frequency of the oscillations of the dipole moment which are induced by the cavity field become less than the frequency of the oscillations of the dipole moment of the dressed states. As follows from Fig. 2b, the coherent and thermal distributions are manifested completely differently. This makes it possible to distinguish these two states of the field easily.

We now note that since every measurement changes the quantum state of the field, even though it leaves unchanged the number $n$ of photons a measurement of the photon statistics on the basis of the mechanism presented above presumes that the previous distribution of the photons is restored after each separate atom is detected. However, if this is not done, then the proposed scheme makes it possible to stimulate a Fock state in a manner similar to Refs. 2 and 3. Figure 3 displays the results of numerical calculations for the collapse of the initial coherent photon distribution with $\langle n \rangle = 7$ into a Fock state with $n = 4$. To simulate the measurements we used the parameters $g_1 = 6 \times 10^4$ s$^{-1}$, $\Omega_0 = 100$ kHz, and $T = 5 \times 10^{-4}$ s$^{-1}$. After the field in the cavity is found to be in a state

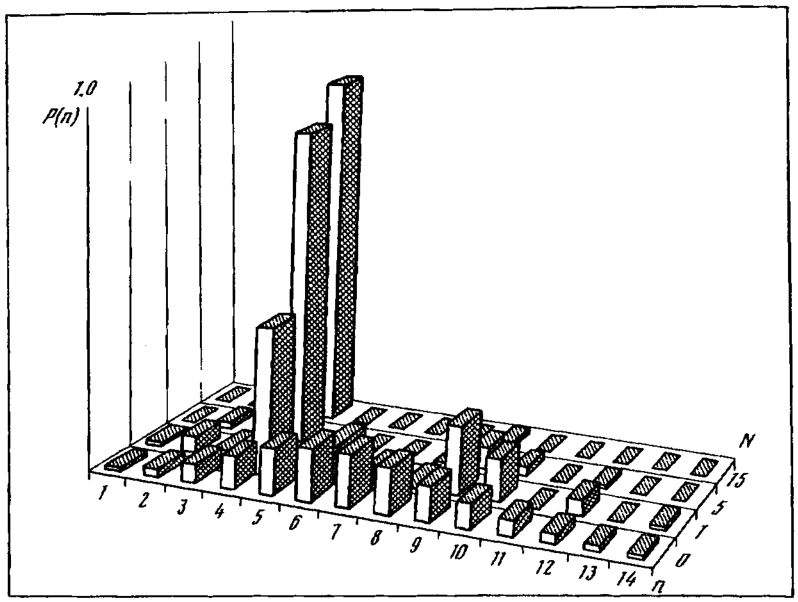


FIG. 3. Distribution $P(n)$ of the number of photons in the cavity, showing the collapse of the initial coherent cavity field with $\langle n \rangle = 7$ into a Fock state with $n = 4$ after successive detection of $N = 1$, 5, and 15 atoms.

with a fixed number of photons, repeated atomic measurements during a time longer than the decay time in the cavity should reveal quantum jumps in the state of the field. These questions will be discussed in detail elsewhere.

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Translated by M. E. Alferieff