Deterministic quantum logic with photons via optically induced photonic band gaps

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We study the giant Kerr nonlinear interaction between two ultraweak optical fields in which the cross-phase-modulation is not accompanied by spectral broadening of the interacting pulses. This regime is realizable in atomic vapors, when a weak probe pulse, upon propagating through the electromagnetically induced transparency (EIT) medium, interacts with a signal pulse that is dynamically trapped in a photonic band gap created by spatially periodic modulation of its EIT resonance. We find that large conditional phase shifts and entanglement between the signal and probe fields can be obtained with this scheme. The attainable π phase shift, accompanied by negligible absorption and quantum noise, is shown to allow a high-fidelity realization of the controlled-phase universal logic gate between two single-photon pulses.

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I. INTRODUCTION

The field of quantum information (QI) is attracting enormous interest, in view of its fundamental nature and its potentially revolutionary applications in cryptography, teleportation, and computing [1]. QI processing schemes rely on the ability to "engineer" and maintain the entanglement of coupled systems. Among the various QI processing schemes of current interest [2–7], those based on photons [6,7] have the advantage of using very robust and versatile carriers of QI. Yet the main impediment toward their operation at the few-photon level is the weakness of photon-photon interaction (optical nonlinearities) in conventional media [8]. One way to circumvent these difficulties is to use linear optical elements, such as beam splitters and phase shifters, in conjunction with single-photon sources and detectors, to achieve *probabilistic* photon-photon entanglement, conditioned on the successful outcome of a measurement performed on auxiliary photons [7].

A promising avenue for *deterministically*, rather than probabilistically, entangling single photons has been opened up by studies of giantly enhanced nonlinear coupling in the regime of electromagnetically induced transparency (EIT) in atomic vapors [9,10]. EIT relies on the classical driving fields to induce coherence between atomic levels and transform the field into an atom-dressed polariton propagating in the medium with controllable, arbitrarily small group velocity [11,12]. These studies have predicted the ability to achieve an appreciable *conditional* phase shift, impressed by one weak field upon another [13], or a two-photon switch [14], using the driven N-shaped configuration of atomic levels. One drawback of these schemes has been the mismatch between the group velocities of the probe pulse moving as a slow EIT polariton and the nearly free propagating signal pulse, which severely limits their effective interaction length and the maximal conditional phase shift [15]. This drawback may be remedied by using an equal mixture of two isotopic species, interacting with two driving fields and an appropriate magnetic field, which would render the group velocities of the two weak pulses equal [16]. Alternative schemes to achieve the group velocity matching and strong nonlinear interaction between the pulses employ a single species of multilevel atoms that couple to both fields in a *symmetric* fashion [17,18].

Notwithstanding its highly promising advantages, deterministic EIT-polariton entanglement faces other serious difficulties. Small group velocities that correspond to long interaction times, and thus large conditional phase shifts, in a medium of finite length (typically of a few centimeters) [16–18] impose limitations on the photonic component of the signal polariton, whose magnitude determines the conditional phase shift. Copropagating pulses pose yet another difficulty: since the conditional phase shift of each pulse is proportional to the local intensity of the other pulse, different parts of the interacting pulses acquire different phase shifts, which causes their frequency chirp and spectral broadening.

As we show here, the foregoing difficulties may be overcome via controlled modification of the photonic density of states in gaseous EIT media, by modulating their refractive index with an off-resonant standing light wave [19]. By properly tuning the resulting photonic band structure, a propagating signal pulse can be converted into a standing-wave polaritonic excitation inside the photonic band gap (PBG). The trapped signal polariton, having an appreciable photonic component, can impress a large, *spatially uniform* phase shift upon the propagating probe, at the single-photon level. The advantageous features of the present scheme pave the way for possible QI applications based on deterministic photon-photon entanglement, without the limitations associated with traveling-wave configurations [15] and without invoking cavity QED techniques [20].

In Sec. II we formulate the basic theory underlying our scheme and give an analytical solution of the equations of motion for the two interacting quantum fields. In Sec. III we

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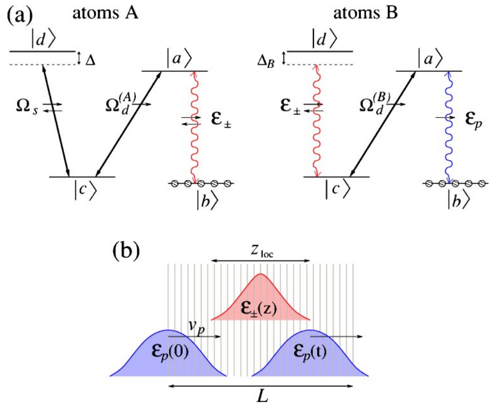

FIG. 1. (Color online) (a) Atomic level scheme involving two species of atoms A and B, aimed at trapping the signal field $\mathcal{E}_\pm$ in a PBG and its cross coupling with the slowly propagating probe field $\mathcal{E}_p$. (b) Probe pulse propagation and interaction with the trapped signal pulse.

study the cross-phase-modulation between the fields while in Sec. IV we discuss an explicit realization of a deterministic controlled-phase (CPHASE) logic gate between two single-photon pulses representing qubits. Our conclusions are summarized in Sec. V.

II. EQUATIONS OF MOTION

We consider a cold atomic medium containing two species of atoms A and B, with N-shaped level configurations [Fig. 1(a)]. Atoms A and B correspond to two isotopic species of trapped alkali-metal atoms subject to an appropriate magnetic field that shifts the Zeeman sublevels and tunes the relevant atomic transitions outlined below. All the atoms are assumed to be optically pumped to the ground states $|b\rangle_{A,B}$. Atoms A and B resonantly interact with two running-wave classical fields driving the atomic transitions $|c\rangle_{A,B}\to|a\rangle_{A,B}$ with the Rabi frequencies $\Omega_d^{(A,B)}$, respectively. In the absence of levels $|d\rangle_{A,B}$, this situation corresponds to the usual EIT for the weak (quantum) signal $\mathcal{E}$ and probe $\mathcal{E}_p$ fields which are acting on the transitions $|b\rangle_{A,B}\to|a\rangle_{A,B}$: In the vicinity of a frequency corresponding to the two-photon Raman resonances $|b\rangle_{A,B}\to|c\rangle_{A,B}$, the medium becomes transparent for both weak fields [9–11]. This transparency is accompanied by a steep variation of the refractive index. Atoms A, in addition, dispersively interact with a standing-wave classical field having the Rabi frequency $\Omega_s(z)=2\Omega_s\cos(k_s z)$ and detuning $\Delta\gg\Omega_s$ from the atomic transition $|c\rangle_A\to|d\rangle_A$. This field induces a spatially periodic ac Stark shift of level $|c\rangle_A$ that results in a spatial modulation of the index of refraction for the signal field according to [19]

$$\delta n=\frac{c}{v_s}\frac{4\Delta_s}{\omega_{ab}}\cos^2(k_s z),$$

where $\Delta_s=\Omega_s^2/\Delta$ is the amplitude of the Stark shift, c/v_s is the ratio of the speed of light in vacuum to the group velocity in the medium, $v_s\propto|\Omega_d^{(A)}|^2$, and $\omega_{\mu\nu}$ is the frequency of the atomic resonance $|\mu\rangle\leftrightarrow|\nu\rangle$. When the modulation depth is sufficiently large, the forward propagating signal field $\mathcal{E}_+$ with a carrier wave vector k near $k_s=w_s/c$ undergoes Bragg scattering into the backward propagating field $\mathcal{E}_-$ with the wave vector $-k$. This scattering of counterpropagating fields into each other forms a standing-wave pattern and modifies the photonic density of states such that a range of frequencies appears in which light propagation is forbidden—a PBG [19]. Both components $\mathcal{E}_\pm$ of the signal field dispersively interact with atoms B via the transition $|c\rangle_B\to|d\rangle_B$ with the detuning Δ_B. Thus atoms of species B simultaneously provide EIT for the slowly propagating probe field $\mathcal{E}_p$ and its cross coupling with the signal field $\mathcal{E}_\pm$ [13,15,16].

We assume that initially the signal pulse of duration T_s enters the EIT medium, where, in the absence of the standing-wave field $\Omega_s=0$, it is slowed down and spatially compressed, by a factor of $v_s^0/c\ll 1$, to the length $z_{\rm loc}\simeq v_s^0 T_s$. Once the signal pulse has been fully accommodated in the medium of length L, which requires that $z_{\rm loc}<L$, it is converted into a standing-wave polaritonic excitation according to the procedure described in [19]. To this end, the driving field $\Omega_{d,0}^{(A)}$, corresponding to the input group velocity $v_s^0\propto|\Omega_{d,0}^{(A)}|^2$, is adiabatically switched off and the pulse is halted in the medium. Next the standing-wave field Ω_s is switched on, thereby establishing the PBG, and finally the driving field is switched back on to a value $\Omega_d^{(A)}>\Omega_{d,0}^{(A)}$, releasing the signal pulse into the PBG. The amplitude of the photonic component of the signal pulse, which is responsible for the cross-phase modulation, is now larger than that at the input by a factor of $\sqrt{v_s/v_s^0}=\Omega_d^{(A)}/\Omega_{d,0}^{(A)}$ [12]. Then, upon propagating through the medium with the group velocity v_p, the probe pulse interacts with both forward and backward components of the signal over its localization length $z_{\rm loc}$ [Fig. 1(b)]. For a large enough product of the signal field intensity $|\mathcal{E}_\pm|^2$ and interaction time L/v_p, both pulses accumulate uniform conditional phase shifts which can exceed π. Finally, the signal pulse is released from the medium by reversing the sequence that resulted in its trapping.

Let us now consider the scheme more quantitatively. To describe the quantum properties of the medium, we use collective slowly varying atomic operators $\hat{\sigma}_{\mu\nu}^{(\iota)}(z,t)=(1/N_\iota^z)\sum_{j=1}^{N_\iota^z}|\mu_j\rangle_{\iota\iota}\langle\nu_j|e^{-i\omega_{\mu\nu}^{(\iota)}t}$, averaged over small but macroscopic volume containing many atoms of species $\iota=A,B$ around position z [11]: $N_{A,B}^z=(N_{A,B}/L)dz\gg 1$, where $N_{A,B}$ is the total number of the corresponding atoms. The quantum radiation is described by the traveling-wave (multimode) electric field operators $\hat{\mathcal{E}}_\pm(z,t)=\sum_q a_\pm^q(t)e^{\pm iqz}$ and $\hat{\mathcal{E}}_p(z,t)=\sum_q a_p^q(t)e^{iqz}$, where a_j^q is the annihilation operator for the field mode with the wave vector k_j+q, k_j being the carrier wave vector of the corresponding field. These single-mode operators possess the standard bosonic commutation relations $[a_i^q,a_j^{q'\dagger}]=\delta_{ij}\delta_{qq'}$, which yield $[\hat{\mathcal{E}}_i(z),\hat{\mathcal{E}}_j^\dagger(z')]=L\delta_{ij}\delta(z-z')$. In a frame rotating with the frequencies of the optical fields, the interaction Hamiltonian has the following form:

$$H=\frac{\hbar N_A}{L}\int dz\{\Delta\hat{\sigma}_{dd}^{(A)}-g_A[\hat{\mathcal{E}}_+e^{ikz}+\hat{\mathcal{E}}_-e^{-ikz}]\hat{\sigma}_{ab}^{(A)}-\Omega_d^{(A)}e^{ik_d^{(A)}z}\hat{\sigma}_{ac}^{(A)}-2\Omega_s\cos(k_sz)\hat{\sigma}_{dc}^{(A)}\}+\frac{\hbar N_B}{L}\int dz\{\Delta_B\hat{\sigma}_{dd}^{(B)}-g_p\hat{\mathcal{E}}_pe^{ik_pz}\hat{\sigma}_{ab}^{(B)}-\Omega_d^{(B)}e^{ik_d^{(B)}z}\hat{\sigma}_{ac}^{(B)}-g_B[\hat{\mathcal{E}}_+e^{ikz}+\hat{\mathcal{E}}_-e^{-ikz}]\hat{\sigma}_{dc}^{(B)}\}+\text{H.c.},\tag{1}$$

where $g_A=\wp_{ab}^{(A)}\sqrt{\omega_{ab}^{(A)}/(2\hbar\epsilon_0SL)}$, $g_p=\wp_{ab}^{(B)}\sqrt{\omega_{ab}^{(B)}/(2\hbar\epsilon_0SL)}$, and $g_B=\wp_{dc}^{(B)}\sqrt{\omega_{dc}^{(B)}/(2\hbar\epsilon_0SL)}$ are the atom-field coupling constants, $\wp_{\mu\nu}^{(\iota)}$ being the corresponding atomic dipole matrix element and S the cross-sectional area of the quantum fields. To facilitate the analysis, we decompose the induced atomic coherences as

$$\hat{\sigma}_{ba}^{(A)}=\hat{\sigma}_{ba}^{+(A)}e^{ikz}+\hat{\sigma}_{ba}^{-(A)}e^{-ikz},$$

$$\hat{\sigma}_{bc}^{(A)}=\hat{\sigma}_{bc}^{+(A)}e^{i(k-k_d)z}+\hat{\sigma}_{bc}^{-(A)}e^{-i(k+k_d)z},$$

$$\hat{\sigma}_{cd}^{(B)}=\hat{\sigma}_{cd}^{+(B)}e^{ikz}+\hat{\sigma}_{cd}^{-(B)}e^{-ikz},$$

and make the transformations

$$\hat{\sigma}_{ba}^{(B)}\to\hat{\sigma}_{ba}^{(B)}e^{ik_pz},\quad \hat{\sigma}_{bc}^{(B)}\to\hat{\sigma}_{bc}^{(B)}e^{i(k_p-k_d)z}.$$

Using the slowly varying envelope approximation, we have the following equations of motion for the weak quantum fields:

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_p(z,t)=ig_pN_B\hat{\sigma}_{ba}^{(B)},\tag{2a}$$

$$\left(\frac{\partial}{\partial t}\pm c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}}_\pm(z,t)=ig_AN_A\hat{\sigma}_{ba}^{\pm(A)}+ig_BN_B\hat{\sigma}_{cd}^{\pm(B)}.\tag{2b}$$

The evolution of the atomic operators is governed by the Heisenberg-Langevin equations [10], which are treated perturbatively in the small parameters $g\hat{\mathcal{E}}/\Omega_d$ and in the adiabatic approximation for all the fields [11],

$$\hat{\sigma}_{ba}^{\pm(A)}=-\frac{i}{\Omega_d^{(A)}}\left[\left(\frac{\partial}{\partial t}+\gamma_A-2i\Delta_s\right)\hat{\sigma}_{bc}^{\pm(A)}-i\Delta_s\hat{\sigma}_{bc}^{\mp(A)}e^{\pm2i\delta kz}\right]+\frac{i}{\Omega_d^{(A)}}\hat{F}_{bc}^{\pm(A)},\tag{3a}$$

$$\hat{\sigma}_{bc}^{\pm(A)}=-\frac{g_A\hat{\mathcal{E}}_\pm}{\Omega_d^{(A)}}-i\frac{\hat{F}_{ba}^{\pm(A)}}{\Omega_d^{(A)}},\tag{3b}$$

$$\hat{\sigma}_{ba}^{(B)}=-\frac{i}{\Omega_d^{(B)}}\left[\left(\frac{\partial}{\partial t}+\gamma_B\right)\hat{\sigma}_{bc}^{(B)}-i\frac{g_B^2(\hat{\mathcal{E}}_+^\dagger\hat{\mathcal{E}}_++\hat{\mathcal{E}}_-^\dagger\hat{\mathcal{E}}_-)}{\Delta_B}\hat{\sigma}_{bc}^{(B)}\right]+\frac{i}{\Omega_d^{(B)}}\hat{F}_{bc}^{(B)},\tag{3c}$$

$$\hat{\sigma}_{bc}^{(B)}=-\frac{g_p\hat{\mathcal{E}}_p}{\Omega_d^{(B)}}+i\frac{\hat{F}_{ba}^{(B)}}{\Omega_d^{(B)}},\tag{3d}$$

$$\hat{\sigma}_{cd}^{\pm(B)}=\frac{g_B\hat{\mathcal{E}}_\pm}{\Delta_B}\hat{\sigma}_{cc}^{(B)}=\frac{g_B}{\Delta_B}\frac{g_p^2(\hat{\mathcal{E}}_p^\dagger\hat{\mathcal{E}}_p)}{|\Omega_d^{(B)}|^2}\hat{\mathcal{E}}_\pm,\tag{3e}$$

where $\delta k=k_s-k$ is the phase mismatch, $\gamma_A=\gamma_{bc}^{(A)}+\gamma_d^{(A)}\Delta_s/\Delta$ and $\gamma_B=\gamma_{bc}^{(B)}$ are the Raman coherence relaxation rates, $\gamma_d^{(\iota)}$ is the spontaneous decay rate of state $|d\rangle_\iota$, and $F_{\mu\nu}^{(\iota)}$ are δ-correlated Langevin noise operators associated with the relaxation.

To solve the coupled set of Eqs. (2a), (2b), and (3a)–(3e), we introduce new quantum fields $\hat{\Psi}_p$ and $\hat{\Psi}_\pm$ (dark-state polaritons [11]) via the canonical transformations

$$\hat{\Psi}_p=\cos\theta_B\hat{\mathcal{E}}_p-\sin\theta_B\sqrt{N_B}\hat{\sigma}_{bc}^{(B)},\tag{4a}$$

$$\hat{\Psi}_\pm=\cos\theta_A\hat{\mathcal{E}}_\pm-\sin\theta_A\sqrt{N_A}\hat{\sigma}_{bc}^{\pm(A)},\tag{4b}$$

where the mixing angles $\theta_{A,B}$ are defined as $\tan\theta_{A,B}=g_{A,p}\sqrt{N_{A,B}}/\Omega_d^{(A,B)}$. It follows from Eqs. (3b) and (3d) that

$$\hat{\Psi}_p=\frac{\hat{\mathcal{E}}_p}{\cos\theta_B}=-\frac{\sqrt{N_B}\hat{\sigma}_{bc}^{(B)}}{\sin\theta_B},$$

$$\hat{\Psi}_\pm=\frac{\hat{\mathcal{E}}_\pm}{\cos\theta_A}=-\frac{\sqrt{N_A}\hat{\sigma}_{bc}^{\pm(A)}}{\sin\theta_A},$$

i.e., inside the medium the photonic component of each polariton is proportional to $\cos\theta_\iota$ while the atomic component to $\sin\theta_\iota$ of the corresponding mixing angle θ_ι. From Eqs. (2a), (2b), and (3a)–(3e), the equations of motion for polaritons are then obtained as

$$\left(\frac{\partial}{\partial t}+v_p\frac{\partial}{\partial z}\right)\hat{\Psi}_p=-\kappa_p\hat{\Psi}_p+i\eta\hat{I}_s\hat{\Psi}_p+\hat{\mathcal{F}}_p,\tag{5a}$$

$$\left(\frac{\partial}{\partial t}\pm v_s\frac{\partial}{\partial z}\right)\hat{\Psi}_\pm=-\kappa_s\hat{\Psi}_\pm+i\eta\hat{I}_p\hat{\Psi}_\pm+i\beta\hat{\Psi}_\mp+\hat{\mathcal{F}}_s,\tag{5b}$$

where $v_p=c\cos^2\theta_B$ and $v_s=c\cos^2\theta_A$ are the group velocities, $\hat{I}_p\equiv\hat{\Psi}_p^\dagger\hat{\Psi}_p$ and $\hat{I}_s\equiv\hat{\Psi}_+^\dagger\hat{\Psi}_++\hat{\Psi}_-^\dagger\hat{\Psi}_-$ the intensity (excitation-number) operators for the probe and signal polaritons, respectively, $\kappa_{s,p}=\gamma_{A,B}\sin^2\theta_{A,B}$ the absorption rates, $\hat{\mathcal{F}}_{s,p}$ the associated δ-correlated noise operators, $\eta=\cos^2\theta_A\sin^2\theta_Bg_B^2/\Delta_B$ the cross-phase-modulation rate between the polaritons, and $\beta=\Delta_s\sin^2\theta_A$ the coupling rate of the forward and backward propagating components of the signal polariton. In Eq. (5b), the linear phase modulation has been absorbed in the signal polariton via the unitary transformation $\hat{\Psi}_\pm\to\hat{\Psi}_\pm e^{2i\beta t}$, and we have assumed that the effective phase matching condition $2\delta kz\ll1$ remains satisfied for $0\leqslant z\leqslant L$ [19]. We have also assumed that the cross absorption is negligible, which requires that $\Delta_B\gg\gamma_d^{(B)}$ [13,14]. Then the cross-phase-modulation η is purely real and is proportional to the intensity of the photonic component of the signal polariton, $\cos^2\theta_A=\hat{\mathcal{E}}_\pm^\dagger\hat{\mathcal{E}}_\pm/(\hat{\Psi}_\pm^\dagger\hat{\Psi}_\pm)$, multiplied by the intensity of the atomic component of the probe polariton, $\sin^2\theta_B=N_B\hat{\sigma}_{cc}^{(B)}/(\hat{\Psi}_p^\dagger\hat{\Psi}_p)$.

Equations (5a) and (5b) are similar to the corresponding equations derived for the case of cross-phase-modulation in a doped photonic crystal [21]. Their general analytical solution for arbitrary initial and boundary conditions of the traveling-wave quantized fields $\hat{\Psi}_{p,\pm}$ is not known. However, when the absorption is small enough to be neglected (see below), for a given time and space dependence of the signal-polariton intensity $\hat{I}_s(z,t)$, the solution for the probe is

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)\exp\left[i\frac{\eta}{v_p}\int_0^z \hat{I}_s(z',\tau + z'/v_p)dz' \right], \quad (6)$$

where $\tau = t - z/v_p$ is the retarded time. An analytic solution for the two counterpropagating components of the signal polariton can be obtained only in the case when the spatial dependence of the probe-polariton intensity can be neglected on the scale of z_{loc}, $\hat{I}_p(z,t) \simeq \hat{I}_p(t)$. This requires that $v_p T_p > z_{\mathrm{loc}}$, where T_p is the duration of the probe pulse. Alternatively, the spectral width of the probe $\delta\omega_p \sim T_p^{-1}$ should satisfy $\delta\omega_p < v_p/z_{\mathrm{loc}}$. Then Eq. (5b) can be solved using the Fourier transform technique [19]. The solution for the polariton-mode operators $\hat{\psi}_\pm^q(t) = \int dz\, e^{\mp iqz}\hat{\Psi}_\pm(z,t)$ is given by

$$\hat{\psi}_+^q(t) = \hat{\psi}_+^q(0)e^{i\hat{\phi}_s(t)}\left[\cos(\chi t) - i\frac{qv_g}{\chi}\sin(\chi t) \right], \quad (7a)$$

$$\hat{\psi}_-^q(t) = i\hat{\psi}_+^{-q}(0)e^{i\hat{\phi}_s(t)}\frac{\beta}{\chi}\sin(\chi t), \quad (7b)$$

where $\chi = \sqrt{q^2 v_s^2 + \beta^2}$. Note that all the spatial modes $\hat{\psi}_\pm^q$ of the signal polariton acquire the same q-independent phase shift $\hat{\phi}_s(t) = \eta\int_0^t \hat{I}_p(t')dt'$, with $\hat{I}_p(t) = \hat{\Psi}_p^\dagger(0,\tau)\hat{\Psi}_p(0,\tau)$. It follows from Eqs. (7a) and (7b) that a signal pulse containing only the modes with $|q| \ll \beta/v_g$ will be strongly trapped inside the medium, its wave packet periodically cycling between the forward and backward components while interacting with the probe polariton. We then obtain

$$\hat{\Psi}_+(z,t) = \hat{\Psi}_+(z,0)e^{i\hat{\phi}_s(t)}\cos(\beta t), \quad (8a)$$

$$\hat{\Psi}_-(z,t) = i\hat{\Psi}_+(z,0)e^{i\hat{\phi}_s(t)}\sin(\beta t), \quad (8b)$$

$$\hat{\Psi}_p(z,t) = \hat{\Psi}_p(0,\tau)e^{i\hat{\phi}_p(z)}, \quad (8c)$$

where $\hat{\phi}_p(z) = (\eta/v_p)\int_0^z \hat{I}_s(z')dz'$, with $\hat{I}_s(z) = \hat{\Psi}_+^\dagger(z,0)\hat{\Psi}_+(z,0)$, is the probe phase-shift operator.

Equations (8a)–(8c) are our central result. Let us dwell upon the approximations involved in the derivation of this solution. During the conversion of the signal pulse into a standing-wave polaritonic excitation inside the PBG, the nonadiabatic corrections resulting in its dissipation are negligible provided the medium is optically thick [11],

$$\frac{g_A^2 N_A z_{\mathrm{loc}}}{c\gamma_a^{(A)}} \simeq \varsigma_A \rho_A L \gg 1,$$

where $\varsigma_A = \wp_{ab}^{(A)^2}\omega_{ab}^{(A)}/(2\hbar\epsilon_0 c\gamma_a^{(A)})$ is the resonant absorption cross section for the transition $|b\rangle_A \to |a\rangle_A$ and $\rho_A = N_A/(SL)$ is the density of atoms A. Due to nonzero values of q, the trapped signal pulse spreads and eventually leaks out of the medium at a rate

$$\kappa_l \simeq \frac{q^2 v_s^2}{\pi\beta}, \quad 0 \leq |q| < \beta/v_s.$$

We can estimate the bandwidth of the signal pulse from its spatial extent as $\delta q \sim v_s/(cL) < |\Omega_d^{(A)}|^2/(\gamma_a^{(A)}c)$, thus obtaining the upper limit for the leakage rate

$$\kappa_l \leq \frac{v_s^4}{\pi c^2 \beta L^2}. \quad (9)$$

Hence, the interaction time $t_{\mathrm{int}} = L/v_p$ is limited by $t_{\mathrm{int}} \times \max\{\kappa_s, \kappa_p, \kappa_l\} \ll 1$. The corresponding fidelity of the cross-phase modulation is given by

$$F = \exp[-(\kappa_s + \kappa_p + \kappa_l)L/v_p]. \quad (10)$$

Thus, to minimize the standing-wave field-induced absorption of the signal polariton, due to the enhanced relaxation of Raman coherence γ_A and index modulation exceeding the transparency window, the ac Stark shift should be limited by

$$\Delta_s < \frac{\gamma_{bc}^{(A)}\Delta}{\gamma_d^{(A)}}, \frac{|\Omega_d^{(A)}|^2}{\gamma_a^{(A)}}. \quad (11)$$

At the same time, the bandwidth of the probe is limited by the length of the medium [22],

$$\delta\omega_p < \frac{|\Omega_d^{(B)}|^2}{\sqrt{g_p^2 N_B \gamma_a^{(B)} L/c}} = \frac{|\Omega_d^{(B)}|^2 k_p}{\gamma_a^{(B)}\sqrt{(3\pi/2)\rho_B L}}, \quad (12)$$

where $\rho_B = N_B/(SL)$ is the density of atoms B.

Under these conditions, as can be deduced from Eqs. (5a), (5b), and (8a)–(8c), the time evolution of the system is described by the effective interaction Hamiltonian

$$H_{\mathrm{eff}} = -\frac{\hbar}{L}\int dz[\eta\hat{\Psi}_p^\dagger\hat{\Psi}_p(\hat{\Psi}_+^\dagger\hat{\Psi}_+ + \hat{\Psi}_-^\dagger\hat{\Psi}_-) + \beta(\hat{\Psi}_+^\dagger\hat{\Psi}_- + \hat{\Psi}_-^\dagger\hat{\Psi}_+)]. \quad (13)$$

Its first term is responsible for the cross-phase-modulation between the probe and signal polaritons, while the second term describes the scattering between the forward and backward components of the signal polariton into each other. Since the probe polariton propagates with the group velocity v_p, the implicit time dependence of the effective Hamiltonian (13) is contained in the probe polariton operators as $\hat{\Psi}_p = \hat{\Psi}_p(z-\zeta)$, where $\zeta = v_p t$. Employing the plane-wave decomposition of the polariton operators

$$\hat{\Psi}_p(z) = \sum_q \hat{\psi}_p^q e^{iqz}, \quad (14a)$$

$$\hat{\Psi}_\pm(z) = \sum_q \hat{\psi}_\pm^q e^{\pm iqz}, \quad (14b)$$

where the mode operators $\hat{\psi}_i^q$ obey, to a good approximation [11], the bosonic commutation relations $[\hat{\psi}_i^q, \hat{\psi}_j^{q'\dagger}] \simeq \delta_{ij}\delta_{qq'}$,

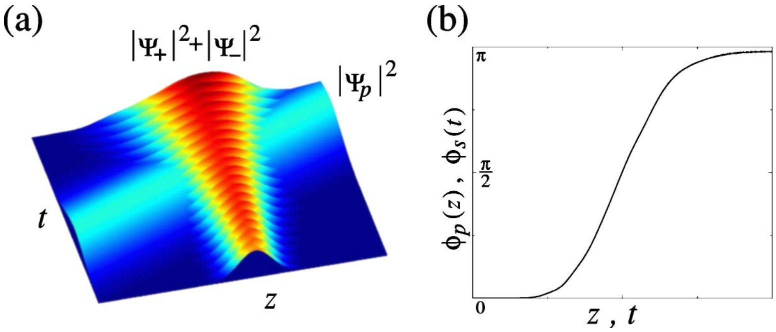

FIG. 2. (Color online) (a) Probe-polariton propagation and interaction with the trapped signal polariton. (b) Probe (signal) phase shift $\phi_p(z) \equiv \phi_p(z, t_{\max}+z/v_p)$ $[\phi_s(t) \equiv \phi_s(z,t)]$ as a function of $z[t]$.

we have $[\hat{\Psi}_i(z), \hat{\Psi}_j^\dagger(z')] \simeq L\delta_{ij}\delta(z-z')$. It is then easy to show that the first and second terms of the effective Hamiltonian (13) commute.

III. CROSS-PHASE-MODULATION

In this section we study the nonlinear interaction between the signal and probe polaritons by exploring the classical as well as fully quantum treatments of the system.

A. Classical fields

We begin with the classical limit of the theory, in which the operators $\hat{\Psi}_{p,\pm}$ and $\hat{I}_{p,s}$ are replaced by the corresponding c numbers. Let us consider two single-photon pulses, which, upon entering the medium, are converted into two polaritons, each containing a single excitation,

$$\frac{1}{L}\int I_s dz = \frac{v_p}{L}\int I_p dt \simeq 1. \qquad (15)$$

Then the conditional phase shifts, accumulated by the probe and signal pulses during the interaction, are given by

$$\phi_p = \phi_s = \frac{\eta L}{v_p} = \frac{g_B^2 L \cos^2\theta_A \tan^2\theta_B}{c\Delta_B} \equiv \phi. \qquad (16)$$

We note again that the phase shift is proportional to the intensity of the photonic component of the signal polariton, as attested by the presence of the $\cos^2\theta_A$ term in the numerator of Eq. (16).

Expressing the atom-field coupling constants g through the decay rate γ of the corresponding excited state as

$$g = \frac{3\pi c\gamma}{2k^2 SL},$$

and assuming that $\gamma_a^{(A)} \simeq \gamma_a^{(B)}$ and $g_A^2 N_A \gg |\Omega_d^{(A)}|^2$ $(v_s \ll c)$, from Eq. (16) we have

$$\phi \simeq \frac{3\pi\gamma_d |\Omega_d^{(A)}|^2 \rho_B}{2k_p^2 \Delta_B |\Omega_d^{(B)}|^2 S\rho_A}. \qquad (17)$$

For realistic experimental parameters, relevant to a cold atomic gas ($T \lesssim 1$ mK) with $L \simeq 1$ cm, $\rho_{A,B} \simeq 10^{12}$ cm^{-3}, $S \simeq 10^{-8}$ cm^2, $\omega_{p,s} \simeq 3\times 10^{15}$ rad/s, $\Omega_d^{(A)} \simeq 5\times 10^8$ rad/s, $\Omega_d^{(B)} \simeq 2\times 10^7$ rad/s, $\Delta_B \simeq \Delta_s \simeq 10^8$ rad/s, $\gamma_{a,d} \simeq 10^7$ s^{-1}, and $\gamma_{bc} \simeq 10^3$ s^{-1}, we obtain $\phi \simeq \pi$ with the fidelity $\mathcal{F} \geqslant 0.98$, the main limiting factor being the collisional relaxation of Raman coherence. In Fig. 2 we show the results of our numerical simulations of the probe polariton propagation and interaction with the trapped signal polariton. One can see in Fig. 2(a) that the trapped signal polariton slowly spreads with the rate κ_l and simultaneously undergoes rapid spatiotemporal oscillations, whose period is determined by the reflection rate $T_{\rm osc} = \pi/\beta$, while the spatial amplitude is given by the penetration depth $\pi v_s/(2\beta) = \pi c/(2\Delta_s \tan^2\theta_A)$ which is much smaller than its spatial extent $z_{\rm loc}$. The resulting phase shifts of the probe and signal pulses are shown in Fig. 2(b).

B. Quantum fields: The evolution operator

We now turn to the fully quantum treatment of the system. Given an input state of the probe and signal polaritons $|\Phi_{\rm in}\rangle$, the state of the system subject to the effective interaction Hamiltonian $H_{\rm eff}$ evolves according to

$$|\Phi(t)\rangle = U(t)|\Phi_{\rm in}\rangle, \qquad (18)$$

where the evolution operator $U(t)$ is defined via

$$U(t) = \exp\left(-\frac{i}{\hbar}\int_0^t H_{\rm eff} dt'\right). \qquad (19)$$

We are interested in the output state of the system at time $t_{\rm out} > L/v_p$, when the probe pulse has left the active medium. Since the first term of Hamiltonian (13), which is responsible for the cross-phase-modulation between the probe and signal polaritons, commutes with the second term, which describes the scattering between the forward and backward components of the signal polariton, the evolution operator (19) can be factorized into the product of two commuting operators $U_{\rm I}$ and $U_{\rm II}$, corresponding to the respective terms of the Hamiltonian,

$$U_{\rm out} = U_{\rm I} U_{\rm II}. \qquad (20)$$

Using the plane-wave decompositions for the polariton operators, Eqs. (14a) and (14b), and recalling that in Eq. (13) $\hat{\Psi}_p = \hat{\Psi}_p(z-\zeta)$ with $\zeta = v_p t$, we have

$$U_{\rm I} = \prod_{q_p} \exp\left[i\phi \hat{\psi}_p^{q_p\dagger}\hat{\psi}_p^{q_p}\sum_q (\hat{\psi}_+^{q\dagger}\hat{\psi}_+^q + \hat{\psi}_-^{q\dagger}\hat{\psi}_-^q)\right], \qquad (21a)$$

$$U_{\rm II} = \prod_q \exp[i\beta t_{\rm out}(\hat{\psi}_+^{q\dagger}\hat{\psi}_-^{-q} + \hat{\psi}_-^{-q\dagger}\hat{\psi}_+^q)]. \qquad (21b)$$

Note that $U_{\rm I}$ does not contain time explicitly, because for $t_{\rm out} > L/v_p$ the cross-phase-modulation is over, as the probe pulse has already left the medium. However, the interaction time L/v_p is contained implicitly in $\phi=\eta(L/v_p)$. Below we will employ the evolution operator of Eqs. (20), (21a), and (21b) to calculate the output state of the system for the single-photon and coherent input states.

C. Single-photon states

Consider first the evolution of two single-photon input pulses, which in the medium correspond to the initial state

$$|\Phi_{\text{in}}\rangle = |1_p\rangle \otimes |1_+\rangle \otimes |0_-\rangle, \tag{22}$$

consisting of two single-excitation polariton wave packets

$$|1_p\rangle = \sum_{q_p} \xi_p^{q_p}|1_p^{q_p}\rangle, \quad |1_+\rangle = \sum_q \xi_+^q|1_+^q\rangle,$$

where $|1_p^{q_p}\rangle = \hat{\psi}_p^{q_p\dagger}|0\rangle$ and $|1_+^q\rangle = \hat{\psi}_+^{q\dagger}|0\rangle$. The Fourier amplitudes $\xi_{p,+}^q$, normalized as $\Sigma_q|\xi_{p,+}^q|^2=1$, define the spatial envelopes $f_{p,+}(z)=\Sigma_q\xi_{p,+}^q e^{iqz}$ of the probe and forward signal pulses that initially (at $t=0$) are localized around $z=0$ and $z=L/2$, respectively [see Fig. 2(a)]. After the interaction, at time $t_{\text{out}}>L/v_p$, the output state of the system is found to be

$$\begin{aligned}|\Phi_{\text{out}}\rangle = U_{\text{out}}|\Phi_{\text{in}}\rangle = e^{i\phi}|1_p\rangle \otimes [\cos(\beta t_{\text{out}})|1_+\rangle \otimes |0_-\rangle \\ + i\sin(\beta t_{\text{out}})|0_+\rangle \otimes |1_-\rangle],\end{aligned} \tag{23}$$

where $|1_-\rangle \equiv \Sigma_q\xi_+^q|1_-^{-q}\rangle$ with $|1_-^{-q}\rangle = \hat{\psi}_-^{-q\dagger}|0\rangle$. Thus, while the signal pulse periodically cycles between the forward and backward modes, the combined state of the system acquires an overall conditional phase shift $\phi=\eta L/v_p$. When $\phi=\pi$ and t_{out} is such that $\beta t_{\text{out}}=2\pi n$ (n being any integer), the output state of the two photons is given by

$$|\Phi_{\text{out}}\rangle = -|\Phi_{\text{in}}\rangle, \tag{24}$$

which can be used to realize a deterministic controlled-phase (CPHASE) logic gate between the two photons representing qubits, as described in Sec. IV.

D. Multimode coherent states

Consider finally the evolution of input wave packets composed of the multimode coherent states

$$|\alpha_p\rangle = \prod_{q_p}|\alpha_p^{q_p}\rangle, \quad |\alpha_+\rangle = \prod_q|\alpha_+^q\rangle, \quad |0_-\rangle = \prod_q|0_-^q\rangle. \tag{25}$$

The states $|\alpha_p\rangle$ and $|\alpha_+\rangle$ are the eigenstates of the input operators $\hat{\Psi}_p(0,t)$ and $\hat{\Psi}_+(z,0)$ with the corresponding eigenvalues

$$\alpha_p(t) = \sum_{q_p}\alpha_p^{q_p}e^{-iq_pct}, \quad \alpha_+(z) = \sum_q \alpha_+^q e^{iqz}. \tag{26}$$

From the operator solutions (8), the expectation values for the polariton operators are then obtained as

$$\langle\hat{\Psi}_p(z,t)\rangle = \alpha_p(\tau)\exp\left[\frac{e^{i\phi}-1}{L}\int_0^z|\alpha_+(z')|^2dz'\right], \tag{27a}$$

$$\langle\hat{\Psi}_\pm(z,t)\rangle = \alpha_+(z)\exp\left[\frac{e^{i\phi v_p/c}-1}{L}c\int_0^t|\alpha_p(\tau')|^2dt'\right] \times \begin{bmatrix}\cos(\beta t)\\ i\sin(\beta t)\end{bmatrix}, \tag{27b}$$

where ϕ is given in Eq. (16). These equations are notably different from those obtained for single-mode [23] and multimode copropagating fields [16,18] because all parts of the probe pulse interact with the whole signal pulse (and the other way around), which is reflected in the space (time) integration. Similarly to the cases discussed in Refs. [16,18,23], only in the limit $\phi \ll 1$ do Eqs. (27a) and (27b) reproduce the classical result,

$$\phi_p = \frac{\phi}{L}\int|\alpha_+|^2dz' = \frac{\eta}{v_p}\int I_s(z')dz', \tag{28a}$$

$$\phi_s = \frac{\phi v_p}{L}\int|\alpha_p|^2dt' = \eta\int_0^t I_p(t')dt', \tag{28b}$$

whereby a phase shift of π can be obtained when

$$\frac{1}{v_p}\int|\alpha_+|^2dz' = \int|\alpha_p|^2dt' = \frac{\pi}{\eta}.$$

This restriction on the classical correspondence of the coherent states comes about since, for large enough cross-phase-modulation rates η, these states exhibit periodic collapses and revivals as ϕ and $\phi v_p/c$ change from 0 to 2π. This fact severely limits the usefulness of weak coherent states for QI applications based on the polarization degrees of freedom of optical fields.

Let us also calculate the time evolution of the input state

$$|\Phi_{\text{in}}\rangle = |\alpha_p\rangle \otimes |\alpha_+\rangle \otimes |0_-\rangle. \tag{29}$$

Using Eqs. (20), (21a), and (21b), and the fact that

$$\begin{aligned}&\exp[i\beta t(a^\dagger b + b^\dagger a)]|n_a\rangle|0_b\rangle \\ &= \sum_{k=0}^n \binom{n}{k}^{1/2}[\cos(\beta t)]^{n-k}[i\sin(\beta t)]^k|(n-k)_a\rangle|k_b\rangle,\end{aligned} \tag{30}$$

where $a^\dagger, a$ and $b^\dagger, b$ are the bosonic creation and annihilation operators for the corresponding field modes [24], we obtain a rather cumbersome, but nevertheless useful result,

$$\begin{aligned}|\Phi_{\text{out}}\rangle &= U_{\text{out}}|\Phi_{\text{in}}\rangle \\ &= \prod_{q_p} e^{-|\alpha_p^{q_p}|^2/2}\sum_{m_1,m_2,\ldots,m_l}\frac{(\alpha_p^{q_p^1})^{m_1}(\alpha_p^{q_p^2})^{m_2}\cdots(\alpha_p^{q_p^l})^{m_l}}{\sqrt{m_1!\,m_2!\,\cdots m_l!}} \\ &\quad\times|(m_1)_p^{q_p^1}\rangle|(m_2)_p^{q_p^2}\rangle\cdots|(m_l)_p^{q_p^l}\rangle \otimes \prod_q e^{-|\alpha_+^q|^2/2} \\ &\quad\times\sum_n[\alpha_+^q e^{i\phi(m_1+m_2+\cdots+m_l)}]^n \\ &\quad\times\sum_{k=0}^n\frac{[\cos(\beta t_{\text{out}})]^{n-k}[i\sin(\beta t_{\text{out}})]^k}{\sqrt{k!\,(n-k)!}}|(n-k)_+^q\rangle|k_-^{-q}\rangle.\end{aligned} \tag{31}$$

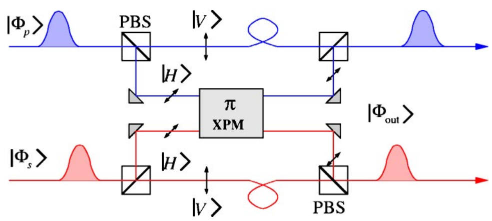

FIG. 3. (Color online) Proposed implementation of the optical CPHASE logic gate between two single-photon qubits, using polarizing beam splitters (PBS's), and π cross-phase-modulation (XPM) studied here.

For $\phi=\pi$, one has $e^{i\phi(m_1+m_2+\cdots+m_l)}=\pm 1$, the "+1" or "−1" corresponding to the sum $(m_1+m_2+\cdots+m_l)$ being, respectively, even or odd. Accordingly, Eq. (31) simplifies to

$$
\begin{aligned}
|\Phi_{\text{out}}\rangle = {}& \frac{1}{2}(|\alpha_p\rangle + |-\alpha_p\rangle) \otimes \prod_q e^{-|\alpha_+^q|^2/2} \sum_n (\alpha_+^q)^n \\
&\times \sum_{k=0}^{n} \frac{[\cos(\beta t_{\text{out}})]^{n-k}[i\sin(\beta t_{\text{out}})]^k}{\sqrt{k!\,(n-k)!}} |(n-k)_+^q\rangle |k_-^q\rangle \\
&+ \frac{1}{2}(|\alpha_p\rangle - |-\alpha_p\rangle) \otimes \prod_q e^{-|\alpha_+^q|^2/2} \sum_n (-\alpha_+^q)^n \\
&\times \sum_{k=0}^{n} \frac{[\cos(\beta t_{\text{out}})]^{n-k}[i\sin(\beta t_{\text{out}})]^k}{\sqrt{k!\,(n-k)!}} |(n-k)_+^q\rangle |k_-^q\rangle,
\end{aligned}
\tag{32}
$$

where $|-\alpha_p\rangle=\Pi_{q_p}|-\alpha_p^{q_p}\rangle$. A particularly simple and important case is realized for $\beta t_{\text{out}}=(\pi/2)n$ (n being any integer), when either $\sin(\beta t_{\text{out}})$ or $\cos(\beta t_{\text{out}})$ is zero and the signal polariton is found in one of the four possible states $|\alpha_+\rangle\otimes|0_-\rangle, |-\alpha_+\rangle\otimes|0_-\rangle, |0_+\rangle\otimes|i\alpha_-\rangle$, and $|0_+\rangle\otimes|-i\alpha_-\rangle$. As an example, when $\beta t_{\text{out}}=2\pi n$, we have

$$
\begin{aligned}
|\Phi_{\text{out}}\rangle = {}& \frac{1}{2}(|\alpha_p\rangle + |-\alpha_p\rangle) \otimes |\alpha_+\rangle \otimes |0_-\rangle \\
&+ \frac{1}{2}(|\alpha_p\rangle - |-\alpha_p\rangle) \otimes |-\alpha_+\rangle \otimes |0_-\rangle,
\end{aligned}
\tag{33}
$$

which is an entangled coherent superposition of macroscopically distinguishable states of two fields. Such entanglement of coherent Schrödinger-cat states [25] can find important applications in schemes of quantum-information processing and communication with continuous variables [26]. Thus, using our scheme, one could contemplate the feasibility of deterministic quantum computation with optical coherent states [27].

IV. DETERMINISTIC LOGIC GATE

Utilizing the scheme of Fig. 3 and the results of Sec. III C, one can realize a transformation corresponding to the CPHASE logic gate between two traveling single-photon pulses representing qubits. To this end, suppose that the qubit basis states $\{|0\rangle,|1\rangle\}$ are represented by the vertical $|V\rangle\equiv|0\rangle$ and horizontal $|H\rangle\equiv|1\rangle$ polarization states of the photon. After passing through a polarizing beam splitter (PBS), the vertically polarized component of each photon is not reflected, while the horizontally polarized component is directed into the active medium. Employing the procedure discussed above, whereby the $|H_s\rangle$ component of the signal pulse is first trapped in the medium, then interacts with the $|H_p\rangle$ component of the probe, and finally is released, the two-photon state $|\Phi_{\text{in}}\rangle=|H_pH_s\rangle$ acquires the conditional phase shift π, as per Eq. (24). At the output, each photon is recombined with its vertically polarized component on another PBS, where the complete temporal overlap of the vertically and horizontally polarized components of each photon is achieved by delaying the $|V\rangle$ wave packet in a fiber loop or sending it though another EIT vapor cell. The resulting transformation corresponds to the truth table of the CPHASE gate,

$$
\begin{aligned}
|V_pV_s\rangle &\to |V_pV_s\rangle, \\
|V_pH_s\rangle &\to |V_pH_s\rangle, \\
|H_pV_s\rangle &\to |H_pV_s\rangle, \\
|H_pH_s\rangle &\to -|H_pH_s\rangle.
\end{aligned}
\tag{34}
$$

Together with the Faraday rotations of photon polarization (implementing arbitrary single-qubit rotations) and linear phase shift, the CPHASE gate is *universal* as it can realize any unitary transformation [1].

V. CONCLUSIONS

In this paper we have proposed a scheme for highly efficient Kerr nonlinear interaction between two weak optical fields. We have shown that large conditional phase shifts and entanglement can be obtained in atomic vapors, in which a weak (quantum) probe pulse, upon propagating through the medium, interacts with a weak signal pulse that is dynamically trapped in a photonic band gap created by spatially periodic modulation of the electromagnetically-induced-transparency resonance. The attainable π phase shift accompanied by negligible absorption and spectral broadening can be used for high-fidelity implementation of the CPHASE universal quantum logic gate between the two single-photon pulses. The proposed scheme may therefore pave the way to quantum-information applications, such as deterministic all-optical quantum computation, dense coding, and teleportation [1].

Before closing, we note that our central equations (5a), (5b), and (8a)–(8c) and consequently the main results Eqs. (23), (27a), and (27b) are similar to those obtained by us for the case of cross-phase-modulation in doped photonic crystals [21], whose practical realization represents a formidable experimental challenge. By contrast, the recent experimental progress in trapping and manipulating light pulses in dynamically controlled photonic band gaps in atomic vapors [28] puts the present scheme within easy experimental reach considering present day technology.

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