Spectral gaps for electromagnetic and scalar waves: Possible explanation for certain differences

M. M. Sigalas and E. N. Economou*
Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011

Maria Kafesaki
*Research Center of Crete–FORTH, P.O. Box 1527, 71110 Heraklion Crete, Greece
and Department of Physics, University of Crete, 714 09 Crete, Greece*
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We study two different scalar wave equations. One of them exhibits the main gross features of the simple scalar and elastic wave propagation in periodic composite media. The other behaves similarly to the electromagnetic waves in preferring the network topology and the higher volume fractions for developing spectral gaps.

There is recently an increased interest in the propagation of classical waves [electromagnetic (EM), acoustic (AC), elastic (EL)] in composite materials, both periodic and random.[1–5] The existence, in periodic media, of an absolute frequency gap where the propagation of waves is forbidden for every incidence direction, can have a profound impact on several scientific and technical disciplines.[1–5] Absolute gaps for EM waves propagating in two- and three-dimensional, periodic, dielectric materials have been investigated both experimentally[3,6–8] and theoretically.[3,9–13] In addition, the spatially localized defect modes that can arise in the vicinity of a perturbation of a periodic dielectric structure have been investigated.[3,4,7] It has also been suggested[3–5] that if the periodic composite material is disordered in such a way that it remains periodic on average, it may be easier to observe in it the localization of classical waves whose frequency is close to an edge of an absolute gap of the corresponding periodic composite material than it would be in a disordered composite material which is homogeneous on average.

In most cases the systems under considerations were binary composites (in many instances air was one of the two components). The low propagation velocity component will be called the scattering component (or material), while the high propagation velocity component will be referred to as the host material. The component is characterized by several parameters. Among them the most important are (i) the propagation velocity ratio c_s/c_h (for EL waves there are three velocity ratios, since each material in general sustains both longitudinal and transverse waves with different propagation velocities); (ii) the volume fraction, $f=V_s/V$, occupied by the scattering material, where V is the total volume of the composite; and (iii) the topology which for our purposes can be classified either as a cermet topology (where the scattering material consists of isolated inclusions each of which is completely surrounded by the host material) or as network topology (where the scattering material is connected and forms a multiple self-intersecting continuous network running throughout the whole composite).

The theoretical and experimental studies especially in the periodic case show that the EM waves behave differently comparing with the simple AC or the EL waves. The two main differences are the following: (i) for simple scalar (acoustic) and elastic waves it seems that the cermet topology is more favorable for spectral gap creation than the network topology;[14–16] the opposite is true for EM waves;[11,12,14,16] (ii) for simple scalar (acoustic) or elastic waves the optimum value of f for gap creation seems to be in the range 0.09 to 0.20, while for EM waves the range of optimum values of f is much broader, sometimes extending to about 0.50 (the optimum value of f depends also on the lattice structure).

One is tempted to connect these differences with the vector character and in particular with the *transverse* vector character of the electromagnetic waves.[14] As a result of these the scattering from a single sphere lacks an isotropic component, which, however, is present in both the scalar and the elastic waves. An isotropic scattering will be reinforced by a close-packed structure, which is consistent with a cermet topology, while a strongly anisotropic scattering may possibly favor the network topology.[14] Another argument offered as a possible explanation for the increased ability of the network topology to create spectral gaps for EM waves is its supposedly unlimited polarizability. However, the polarizability is limited by the wavelength of the radiation which is comparable or smaller than the dimensions of the primitive cell (in the spectral gap regions); as a result of this the polarizability argument does not seem to be convincing.[14]

In this paper we analyze two different scalar wave equations. One of the two exhibits the main characteristics of the EM waves in spite of the fact that it is a scalar equation. This provides strong evidence that the explanation of the different behavior of the EM case is not mainly connected with its transverse vector character, but with the specific mathematical structure of the corresponding differential equation

$$\nabla\times(\epsilon^{-1}\nabla\times\mathbf{H})=(\omega^2/c^2)\mathbf{H}\ . \tag{1}$$

The first scalar equation we have studied is the ordinary simple scalar equation:

$$\nabla^2\phi = -(\omega^2/v^2)\phi \ , \qquad (2)$$

where $v=v_s$ or v_h in the scattering and the host component, respectively, and ω is the (angular) frequency of the wave. The methods of numerical calculation are described in Ref. 14. In Fig. 1(a) we show the threshold ratio $r_c^2=(v_h^2/v_s^2)_c$ (above which a spectral gap opens up) versus the volume fraction f (of the scattering, i.e., the low-velocity component) for the cermet topology, i.e., for isolated spheres of the scattering component forming face centered (fcc) or body centered (bcc) or simple cubic (sc) periodic lattices. We see that the optimum values of f is around 0.20 and that the minimum value of r_c^2 is 3, 3.3, and 5.4 for fcc, bcc, and sc, respectively. As shown in Fig. 1(b) the network topology [where the host (high v) material is now the spheres (overlapping or not, depending on f) and the scattering material occupies the space among the spheres forming a continuous self-intersecting network] is clearly less favorable than the cermet topology of Fig. 1(a). Indeed, the minimum value of r_c^2 is now 4.7 and 6.2 for fcc and sc, respectively.

In Figs. 2(a) and 2(b) we show corresponding results but for the following wave equation:

$$\nabla(v^2\nabla\phi) = -\omega^2\phi \ , \qquad (3)$$

where again $v=v_s, v_h$ for the scattering and host material, respectively. Apart from the vector character, Eq. (3) is similar to the EM equation [Eq. (1)]. We see in Fig. 2(a) that for Eq. (3), the cermet topology is not so favorable for spectral gap appearance, since the minimum required value of r_c^2 is about 9 as opposed to 3 for the cermet case of Eq. (2). On the other hand, the network topology for Eq. (3) produces significantly lower values of r_c^2 than the cermet topology: 4.5, 8 [see Fig. 2(b)] for fcc and bcc as opposed to 9.5 and 9 for the corresponding values of the cermet topology [Fig. 2(a)]. Furthermore, the optimum value for f is considerably higher as compared with those of Eq. (2) but they are comparable with those of EM waves[11,12,14] [Eq. (1)]; in particular for fcc the optimum f are 0.3 and 0.6 for the cermet and network topologies respectively; for bcc structure the optimum f are 0.4 and 0.7 for the cermet and network topologies, respectively. Thus Eq. (3) behaves similarly to the EM equation regarding both its preference for the network topology as well as higher optimum values of f (close to 0.50). The sc structure seems to prefer the cermet topology, since there are not gaps for the network topology, but the r_c^2 in that case are extremely high (greater than 13.5) so it does not seem to change the previous conclusion. Also, notice that for sc structure the cases with

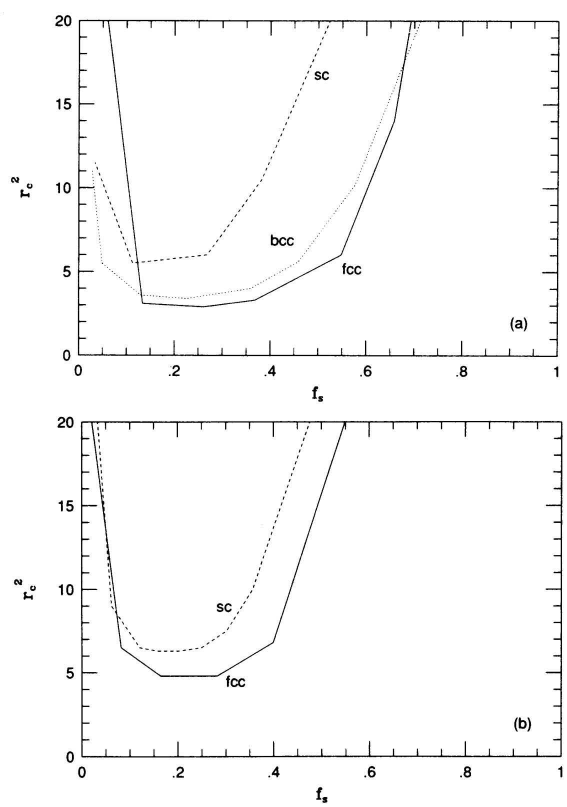

FIG. 1. The threshold velocity ratio $r_c^2=(v_h^2/v_s^2)_c$ for the appearance of a spectral gap vs the volume fraction f of the low velocity (v_s) component for Eq. (2). (a) The v_s component consists of isolated spheres forming face-centered (fcc), body-centered (bcc), and simple cubic (sc) lattices. (b) The c_h component consists of isolated spheres forming fcc or sc lattices.

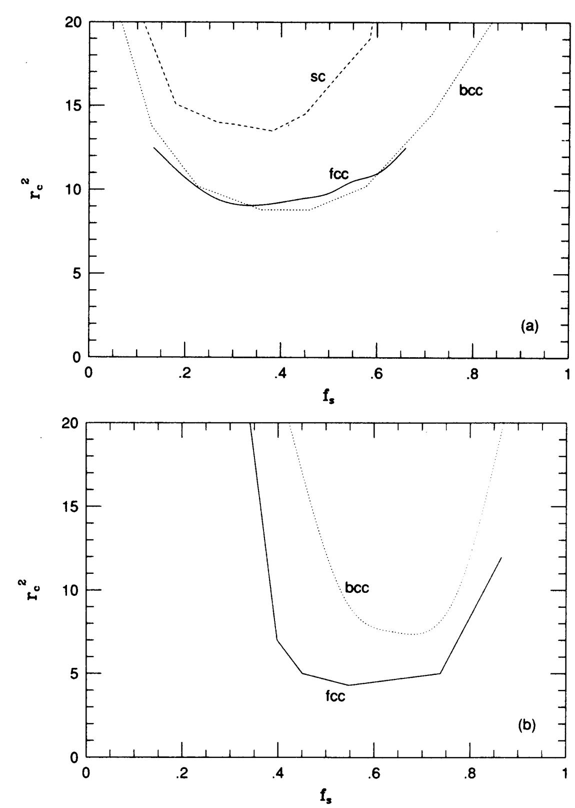

FIG. 2. The threshold velocity ratio r_c^2 vs the volume fraction f of the low velocity (v_s) component for Eq. (3). (a) The v_s component consists of isolated spheres forming fcc, bcc, and sc lattices. (b) The c_h component consists of isolated spheres forming fcc or bcc lattices.

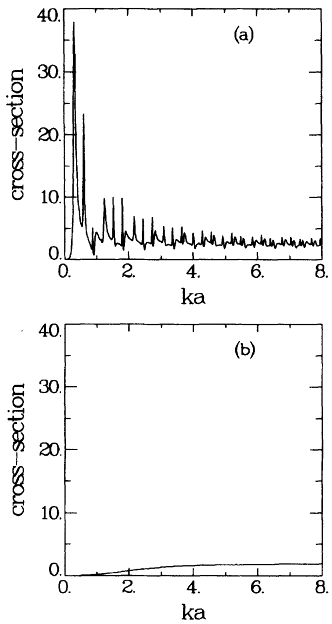

FIG. 3. The total scattering cross section vs ka for a single sphere of radius a and propagation velocity $v_i = v_0/5$ (a) or $5v_0$ where v_0, k are the propagation velocity and the wave number of the surrounding material. The wave is described by Eq. (2).

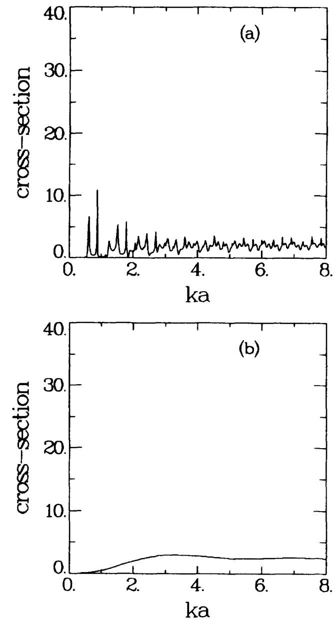

FIG. 4. The same as Fig. 3, but the wave is described by Eq. (3).

f greater than 0.52 are belong to network topology and the optimum $f(=0.40)$ is close to that value. It must be pointed out, that these differences between Eqs. (2) and (3) persist in two-dimensional systems as well.[17]

A clue for the origin of these differences between Eqs. (2) and (3) may be come by considering the scattering from a single sphere embedded in a uniform background with propagation velocity v_0. In Fig. 3 we show the total scattering cross section for Eq. (2) (which corresponds to the boundary conditions on the surface of the sphere, both ϕ and $\partial\phi/\partial r$ continuous). In Fig. 3(a) the propagation velocity inside the sphere $v_i = v_0/5$. This is the single scatterer version of the cermet topology. We see that the cross section is large on the average and in addition exhibits very large resonances. In contrast, for the case where $v_i = 5v_0$ [Fig. 3(b)] the cross section is small without any resonances. The conclusion from this comparison is that much stronger scattering is produced for Eq. (2) if the high-velocity material surrounds the low-velocity material than vice versa. Hence, Eq. (2) would produce spectral gaps (in the periodic case) or localization (in the random case) easier in the case of the cermet topology than in the case of the network topology. Figure 4 shows that Eq. (3) (which corresponds to the boundary conditions, both ϕ and $v^2\partial\phi/\partial r$ continuous) behaves differently: In spite of the complicated resonance structure for the case where $v_i = v_0/5$ [Fig. 4(a)] the cross section on the average is about the same as in the case where $v_i = 5v_0$ [Fig. 4(b)] indicating that there is not any strong dependence on the velocity of the surrounding material. Thus one is led to expect that there is not any advantage to the cermet topology and that a topology consisting of two interpenetrating complicated networks (one for each of the two components) of about equal volume fraction may be closer to the optimum structure for Eq. (3) to develop spectral gaps. Figure 2 as well as the data of Ref. 14 tend to support this suggestion.

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*Permanent address: Research Center of Crete and Department of Physics, University of Crete.

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[17]In Ref. 14 the case of EM wave propagation in a composite system consisting of parallel cylinders embedded in a host material was studied. If the propagation vector is normal to the cylinders (which correspond to a two-dimensional case), the wave equation for the E-polarized wave is described by Eq. (2) and the wave equation for the H-polarized wave is described by Eq. (3).