Wave guides in two-dimensional elastic wave band gap materials

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We study the guiding of elastic waves through linear defect modes created by introducing a line of defects in a two-dimensional elastic wave band gap material. These defect modes can act as waveguides in the frequency regime of the gap. The transmission coefficient through these guides as a function of frequency has been found to exhibit pronounced dips. Here we confirm the existence of these dips, by presenting results for a variety of composites, and also we examine in detail their origin.

Keywords: elastic waves, wave guides, band gaps, defects

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The propagation of classical (electromagnetic, acoustic, elastic) waves (CW) in periodic media has recently received a great deal of attention. This attention stems mainly from the possibility of creating periodic structures which exhibit in their spectrum band gaps, i.e. frequency regions in which the CW propagation is prohibited. These band gap structures can be proved very important for a lot of branches of science and technology (e.g. filters, antennas etc.). Also, as classical periodic media are clean and easily constructed experimental systems, they offer themselves for experimental study of a variety of fundamental physical problems, mainly problems related with the disorder induced localization of the waves.

The electromagnetic wave band gap materials (photonic crystals) have been thoroughly studied both theoretically and experimentally [1–7]. Despite the optimal conditions for the appearance of gaps, the existence of defect modes, surface states and guided waves have been also examined in detail [5–7].

Concerning the elastic waves, we should mention that their full longitudinal and transverse vector character and the variety of parameters (density, longitudinal and transverse velocity) that control their propagation make them rich in physics and thus very interesting system. This variety of parameters has been found that gives the possibility of creation of very wide gaps in a large number of composites [8–14]. Concerning the existing study of periodic composites, although the optimal conditions for the appearance of gaps have been examined in detail, there is a limited study for periodic systems with defects and disorder induced phenomena [15–17]. Within this work we try to do a step to this direction.

Here, we study the elastic wave propagation through linear defect modes formed by producing a line of defects (linear defect) in a two-dimensional (2D) periodic system (a system of cylinders embedded periodically in a host) exhibiting gaps. The linear defect is produced by removing one row of cylinders from the periodic system or by reducing the radii of these cylinders.

For electromagnetic (EM) waves it has been shown that a linear defect produced by removing one row of cylinders from a periodic system exhibiting a gap can support a localized mode and, when the mode frequency lies inside the gap, this defect can act as a waveguide ("linear defect guide") which can propagate EM waves with frequencies in the gap regime almost completely without losses [6]. These "linear defect guides" were found to have very similar transmission characteristics with the conventional electromagnetic guides [18]. For elastic waves, recent first results showed a possibility of great guiding of the waves through the same type of defects [19]. The transmission coefficient (T) through the elastic "linear defect guides" however was found [19] to exhibit pronounced dips for some frequency regimes inside the gap of the periodic system. These dips, which are due to opening of gaps in the guided wave propagation (gaps inside the gap of the periodic system), were attributed (a) to the interference of the "complex" (mixed longitudinal and transverse) guided modes and (b) to the periodicity of the guide "boundaries". The periodicity of the "boundaries" implies a periodic potential in the almost one-dimensional (1D) guided wave propagation and this potential can easily lead to opening of gaps in the spectrum.

Here we confirm the existence of the above mentioned dips in the transmission coefficient of the elastic "linear defect guides" (guides formed as linear defects in elastic wave band gap materials) by examining a variety of composites. Also, we study in more detail the origin and the position of these dips. The last is done through comparison with the transmission of conventional elastic wave guides and also through examination of the defect state formation as one goes from a periodic system to a system with a guide (i.e., as one gradually reduces the radii of one row of scatterers).

The study of the transmission properties is performed by using the Finite Difference Time Domain Method (FDTD). The FDTD is based on the discretization of the time dependent elastic wave equation in both the space and the time domains. Through this discretization one can obtain the displacement field as a function of time at any point of a sample. The field as a function of frequency, and thus the transmission coefficient, is calculated by fast Fourier transforming of the time results. (For the implementation of the FDTD method in this kind of problems see Ref. [20]).

For a more detailed examination of the defect modes we calculate the dispersion relation for the structure with the defects. This is done by using the Plane Wave (PW) method [8,9,14,17] in combination with a supercell scheme [17]. A supercell of the structure of interest is repeated periodically and the dispersion relation is calculated through Fourier transforming of the wave equation.

Taking in to account that the existence of gaps is precondition for the creation of localized modes, we choose systems which have been found to exhibit wide gaps. In this work we present results concerning composites formed by Pb cylinders in PMMA host, W cylinders in PMMA and Ag cylinders in epoxy. In most of the cases, for the calculation of the transmission we consider samples of 7×8 cylinders, placed in a host within a square array. For the FDTD implementation we consider 40 grid points per lattice constant of the periodic structure and for the PW supercell method 1089 plane waves.

We first study a system of Pb cylinders in PMMA. The periodic system Pb cylinders in PMMA (in square arrangement) exhibits a very wide full band gap for a wide range of Pb filling ratios, f, with the widest gap at filling ratio around 0.28 (which corresponds to a ratio cylinder radius, r_c, over lattice constant, a, around 0.3). Using the FDTD method and calculating the transmission coefficient for a periodic Pb in PMMA finite sample, of $r_c/a = 0.3$, one can obtain the result shown in Fig. 1(a) - dashed line. For obtaining the result of Fig. 1(a) we consider as incident wave a longitudinal pulse with a Gaussian envelop in space, propagating in the $\langle 10 \rangle$ direction.

By removing completely one row of cylinders from the periodic sample and by calculating the transmission coefficient through the "guide" formed, we obtain the result of Fig. 1(a) - solid line. We should note that the transmission coefficient is determined by calculating the transmitted energy flux in points one unit cell before the exit of the guide (for the total transmission we average over several points), and by dividing by the energy flux of the incident wave (for the calculation of the incident energy flux we remove completely the sample).

As can be seen in Fig. 1(a), the gap of the periodic system has been replaced by a region of transmission almost equal to unity. The localized mode supported by the linear defect of the missing cylinders has led to almost complete guiding of the wave through the crystal. This guiding of the wave can be visualized very clearly in Fig. 1(b), where we show the field over the sample at a given time point.

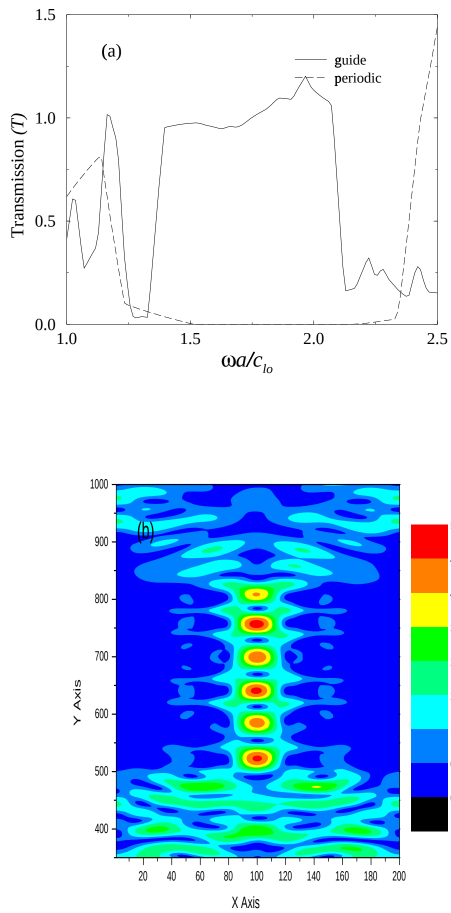

FIG. 1. (a): *Solid line*: Transmission coefficient (T) vs frequency through a guide formed by removing one row of cylinders from a Pb cylinders in PMMA periodic system. For the periodic system the ratio of the cylinders radius over lattice constant is $r_c/a = 0.3$. ω is the frequency and c_{lo} is the longitudinal sound velocity in PMMA. *Dashed line*: The transmission coefficient for the undistorted system for the Pb in PMMA periodic host. (b): The square of the field amplitude over part of the sample described at (a). The incident wave is a monochromatic longitudinal plane wave of frequency $\omega a/c_{lo} = 1.95$. The units in the axes are grid pointsand $a = 40$ grid points.

Close to the upper edge of the gap, one can see however a dramatic drop of the transmission. This drop, which appears also for transverse incident wave, is deep enough and not very close to the upper edge of the gap as to be attributed to leaking of the wave inside the crystal (due to the less localized nature of the guided mode as we approach the ends of the gap). Indeed, as is proved from snapshot pictures like the one of Fig. 1(b), for $\omega a/c_{lo}$ between 2.15 and 2.25 the incident wave is reflected back from the guide (c_{lo} is the longitudinal wave velocity in the host). This is a strong indication for the appearance of gap in the guided wave propagation. The appearance of this gap is confirmed through calculation of the dispersion relation for the system. The dispersion relation result is shown in the right panel of Fig. 2. As has been already discussed, the existence of gaps in the guided wave propagation is not surprising as the propagation is almost 1D and the periodicity of the guide "boundaries" can easily open gaps in the spectrum.

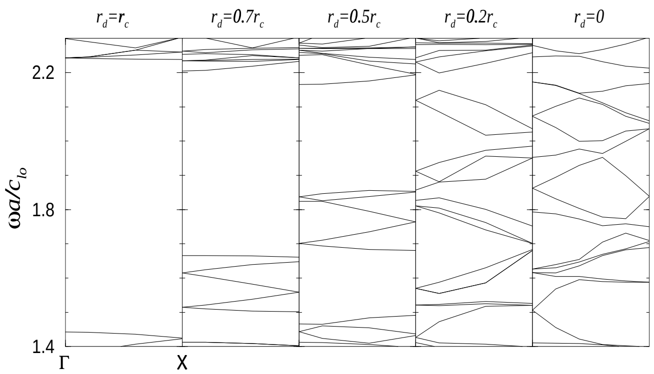

FIG. 2. Dispersion relation along the ΓX axis for a system of Pb cylinders in epoxy with $r_c/a = 0.3$. The pictures show results for a 5×5 supercell of the composite, where the radius, r_d, of the third row of the cylinders is reduced gradually from $r_d = r_c$ (left) to $r_d = 0$ (right), leading from a periodic system to a guide.

It has been also discussed [19] that another parameter possibly responsible for the opening of gaps in the guided wave propagation is the interference of the "complex" (mixed longitudinal and transverse) guided modes. In order to check the importance of this parameter we tried to examine the transmission through an elastic guide with straight, almost impermeable boundaries - "conventional" elastic guide - (by using the FDTD). We formed a conventional guide by replacing the finite periodic crystal by two rectangular slabs of an homogeneous material with very high density. The guide was formed leaving a space between the two slabs. Calculating the transmission for guide widths a and $2a$ we didn't find any regime where the transmission as a function of frequency goes from high to considerably low values. This leads to the conclusion that the dominant parameter for the gaps in the guided modes spectrum is the periodicity of the guide "boundaries", and that the complex character of the elastic modes has minor influence. This is expected if one takes into account that the guided wave motion is almost 1D and thus the mixing of the modes is not strong. (In 1D longitudinal and transverse waves are uncoupled.)

Below we examine in more detail the formation of the gaps in the guided wave propagation through the elastic "linear defect guides". We are interested to see whether these gaps have the trend to appear close to the upper edge of the gap of the periodic system or one can control their position. To examine this, instead of removing a row of cylinders from a periodic system, we gradually reduce the radii of these cylinders and, using the supercell PW method, we calculate the dispersion relation for the resulting system. We consider a supercell of 5 × 5 unit cells where we gradually reduce the radius (r_d) of the central row of cylinders (defect row) from $r_d = r_c$ to $r_d = 0$.

We can see that as we decrease the defects radius the bands of localized states start to appear at the lower edge of the gap and, as the r_d is decreased more, they move upwards. At $r_d = r_c/2$ there is a band of localized states situated in the middle of the gap of the periodic system. Calculating the transmission for this case ($r_d = r_c/2$) by using as incident wave a longitudinal pulse we can see that the localized band does not appear in the transmission coefficient picture. Using however a transverse incident pulse we can see a transmitted frequency regime at exactly the same position as the corresponding bands shown in Fig. 2 - third panel. In this case the guided modes are mainly, but not exclusively, transverse and are not coupled with the longitudinal incident pulse.

As r_d is decreased more, more localized states start to appear from the lower edge of the gap which move upwards. We can see that for $r_d = 0$ there is a small gap remaining between the higher localized band and the upper edge of the gap of the pure periodic system. This small gap, as we mentioned above, is responsible for the abrupt drop of the transmission coefficient shown in Fig. 1(a).

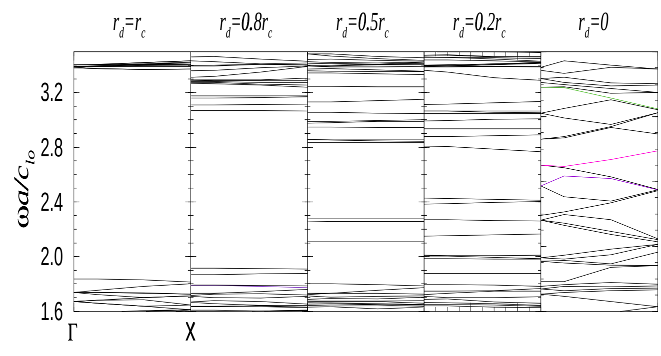

FIG. 3. The same as in Fig. 2 for a system of Ag cylinders in epoxy.

The appearance of the localized modes from the lower edge of the gap as the r_d is decreased is not the only case for the elastic waves. Studying a system of Ag cylinders embedded in epoxy we can see that, as the r_d is decreased, the localized states start to appear from both edges of the gap. For $r_d = 0$ there is a gap in the system of the guide which is situated almost in the middle of the gap of the periodic host. The dispersion relation for a system of Ag cylinders in epoxy where the radii of one row of cylinders are gradually reduced is shown in Fig. 3. For $r_d = 0$ the corresponding transmission coefficient [19], as is expected, exhibits a pronounced dip at the regime of the small gap. This is the case also for W in PMMA (there, the supercell method does not give very accurate results for a 5×5 unit cells supercell, possibly due to the high density contrast between W and PMMA). The transmission coefficient for a system of W in PMMA with $r_c/a = 0.3$ and one row of missing cylinders is shown in Fig. 4.

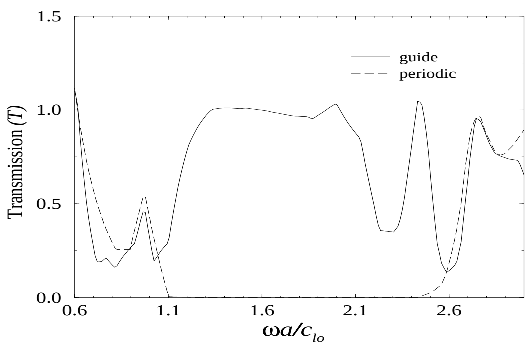

FIG. 4. *Solid line*: Transmission coefficient (T) vs frequency for a guide formed in a periodic system of W cylinders in PMMA with $r_c/a = 0.3$. ω is the frequency, a is the lattice constant and c_{lo} the longitudinal sound velocity in PMMA. *Dashed line*: The transmission coefficient for the periodic host.

Trying to examine if there are certain rules that determine the appearance of localized states as the r_d is decreased, we found that, usually, for scatterers with wave velocities lower or comparable than those of the host the localized states tend to appear from the lower edge of the gap of the periodic system. For scatterers with velocities higher than those of the host the localized states seem to appear from both the edges of the gap. This however needs further examination.

As a conclusion, we can say that periodic elastic materials, due to their possibility to form large gaps, can be used for the creation of very efficient wave guides (elastic "defect guides"). This, however, is not valid at any frequency regime inside the gap, as the transmission through the elastic "linear defect guides" can exhibit pronounced dips at certain frequency regimes. These dips are due to opening of gaps in the guided wave propagation. Responsible factor for these gaps seems to be, mainly, the periodic "boundaries" of the guides, which impose a periodic potential in the propagation of the guided waves.

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