# Negative refraction through an impedance-matched left-handed metamaterial slab

**Koray Aydin and Ekmel Ozbay**

*Nanotechnology Research Center and Department of Physics, Bilkent University, Bilkent 06800, Ankara, Turkey*

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We report the transmission and reflection characteristics of a two-dimensional (2D) left-handed metamaterial (LHM). A well-defined left-handed (LH) transmission band with a peak value of −9.9 dB is obtained at frequencies where both effective permittivity and permeability are negative. A very sharp dip (−38 dB) at the reflection spectrum due to impedance matching at the surface of a 2D LHM is observed. Gaussian beam shifting experiments are performed to study the LH properties of a LHM structure. The structure has a negative refraction of electromagnetic waves in a certain frequency range. The negative refractive index values obtained for four different incident angles are in good agreement. 

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## 1. INTRODUCTION

Recently, novel types of artificially constructed materials, so called left-handed metamaterials (LHMs), have been extensively studied. In his theoretical work,[1] Veselago predicted that it is possible to achieve a negative refractive index with materials that simultaneously have negative values of dielectric permittivity ($\epsilon$) and magnetic permeability ($\mu$). In such media, the electric, magnetic, and wave vector components form a left-handed (LH) coordinate system; hence the name left-handed material is used as a description. Veselago also investigated various interesting optical properties of LHMs such as backward Cherenkov radiation and reversal of the Doppler effect.[1] In ordinary materials, $\epsilon$ and $\mu$ are generally positive. However, artificially constructed periodic structures allow negative values of $\epsilon$ and $\mu$. The periodic arrangement of thin-wire structures are shown theoretically and experimentally to exhibit a plasma frequency at the microwave frequency level.[2,3] These structures behave similarly to a high-pass filter, meaning that the effective permittivity will take on negative values below the plasma frequency. Pendry *et al.* proposed a resonant structure, called a split-ring resonator (SRR), that enhances the electromagnetic response owing to its capacitive elements.[4] When these SRRs are combined in a periodic medium, the magnetic permeability possesses negative values at frequencies close to the magnetic plasma frequency.

The first experimental realization of LHMs was achieved by Smith *et al.*[5] Periodic arrays of SRRs and thin-wire structures were shown to exhibit a LH transmission band at frequencies where both $\epsilon$ and $\mu$ are negative.[5–7] This experiment was later followed up by the direct measurement of the negative index of refraction.[8] Although the negative refraction phenomenon is discussed on the basis of the dispersion of LHMs,[9] further theoretical calculations[10] and experimental studies[11–13] have supported the existence of negative refraction. Negative refraction is also achievable in two-dimensional (2D) dielectric photonic crystals that have a periodically modulated positive permittivity and a permeability of unity.[14–17]

In this paper, we investigate the transmission, reflection, and refraction characteristics of a two-dimensional (2D) LHM. Our paper is organized as follows. First, we present the transmission and reflection spectra of a 2D LHM that exhibits a peak in the transmission spectrum and a dip in the reflection spectrum at LH frequencies. We then verify the negative refraction experimentally by using a method different from that of the wedge structure experiments. The Gaussian beam shifting method at the second LHM–air interface is utilized to measure the refractive index value.

## 2. EXPERIMENT AND ANALYSIS

LHM structures are generally composed of SRRs and thin-wire grids. A periodic thin-wire medium is responsible for the negative effective permittivity, whereas the periodic SRR structure provides negative effective permeability. SRR and wire patterns are fabricated on the front and back sides of FR4 printed circuit boards. The metal used for deposition is copper and has a thickness of 30 $\mu$m. A single SRR cell has a diameter of 7.2 mm, the widths of both rings are 0.9 mm, and the split width and the distance between inner and outer rings are both 0.2 mm.[13] The length and width of the continuous thin-wire structures are $l=19$ cm and $w=0.9$ mm, respectively. Figure 1(a) presents a schematic drawing of a 2D LHM structure. The shaded parts show a single unit cell; two SRRs and the wires are placed perpendicular to each other. The 2D LHM structure is composed of $N_x=5$, $N_y=20$, and $N_z=40$ unit cells, with lattice spacings $a_x=a_y=a_z=9.3$ mm as seen in Fig. 1(b).

Transmission and reflection measurements are performed in free space. The experimental measurement setup consists of a HP 8510C network analyzer and microwave horn antennas. The incident electromagnetic (EM) wave propagates along the $x$ direction, while $\mathbf{E}$ is along the $y$ direction and $\mathbf{H}$ is along the $z$ direction [see Fig. 1(a)]. Transmitter and receiver horn antennas are connected to the HP-8510C network analyzer to measure the transmission coefficients. First we measured the transmission spectrum in free space (i.e., without a LHM structure). This measurement was used as the calibration data for the network analyzer. Then we inserted the structure between the horn antennas, and we performed the transmission measurements by maintaining distance between the fixed transmitter and receiver antennas. The distance between the horn antennas are kept at 35 cm to get rid of near- field effects. For the reflection measurements, two horn antennas were placed close to each other by keeping the angle between the antennas very small. The transmitter horn antenna sends the EM wave to the surface, and the receiver antenna measures the amplitude of the reflected EM waves. For calibration purposes we placed a thick slab of metal (since metals reflect all of the incident EM waves) 12 cm away from the antennas. The metamaterial was also placed at a same distance away from the antennas.

Figure 2 depicts the measured transmission (solid curve) and reflection (dashed curve) spectra of a 2D LHM in the range 3.0–5.5 GHz. The SRR structure is responsible for the negative permeability in the frequency range 3.55–4.05 GHz.[13] The periodic wire medium has a plasma frequency at 8.0 GHz, below which the effective dielectric permittivity takes negative values.[13] It is well known that if there exists a frequency region where effective $\epsilon$ and $\mu$ are both negative, a LH transmission band is likely to occur.[5,6,11,13] In our case, a transmission band is observed

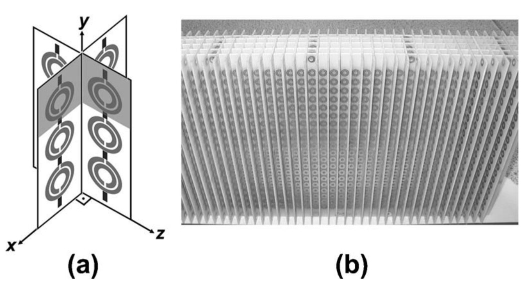


Fig. 1. (a) Schematic drawing of 2D LHM. Shaded parts represent the unit cell of the LHM structure. (b) 2D LHM structure used for transmission, reflection, and refraction experiments.

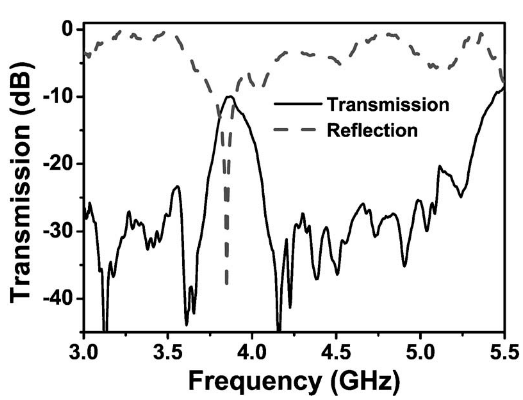


Fig. 2. Measured (a) transmission (solid curve), and (b) reflection (dashed curve) spectra of a 2D LHM structure between 3.0–5.5 GHz.

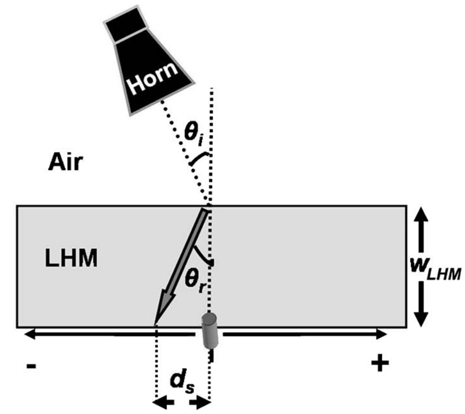


Fig. 3. Schematic drawing of the experimental setup for verifying a negative refractive index. An electromagnetic wave is transmitted to the first interface by a microwave horn antenna with an angle $\theta_i$. The EM wave is refracted with an angle $\theta_r$ through a slab of a LHM structure.

between 3.75–4.05 GHz (Fig. 2). At this frequency range the effective parameters of the material (i.e., $\epsilon$ and $\mu$) possess negative values; therefore the transmission band is indeed left-handed. The transmission peak measures −9.9 dB at 3.86 GHz.

A dip in the reflection spectrum is observed at 3.86 GHz, corresponding to a dip value of −38 dB. At this specific frequency, the impedance is matched to the free space. Therefore, almost all of the EM waves enter the LHM structure without being reflected at the surface.

If the real parts of $\epsilon$ and $\mu$ are equal, the impedance of LHM will be equal to that of free space.[18] As seen in Fig. 2, the impedance-matched frequency region is very narrow. Such a small range for an impedance-matched frequency region is expected, since $\mu$ is known to vary rapidly between the resonance and magnetic plasma frequency, although $\epsilon$ varies slowly.[4,19]

Although the reflection spectra of a SRR-only and a wire-only medium ave been studied previously,[20,21] to our knowledge no experimental evidence of an impedance-matched metamaterial exists. Impedance matching at the surface of a metamaterial is desired, since it reduces the complications of front-face reflection[22] and ensures that the negatively refracted beam is not the result of any experimental artifacts.[23] Much more energy is transferred into the medium at impedance-matched frequencies. Therefore, the higher transmission (−9.9 dB) can be explained by better impedance matching between the free space and the LHM for our particular design. Additionally, the matched impedance at the surface ensures the validity of the previously reported phase shift experiments for the same structures.[13]

Observations of negative refraction through left-handed metamaterials are performed mainly by using wedge-shaped samples.[8,11–13] Furthermore, a phase shift experiment is a way to verify and calculate the negative refractive index.[13] Here we present an alternative way to measure the refractive index of LHMs. The experimental procedure is similar to that of our previous studies on the negative refraction of 2D photonic crystals.[15,17]

The refraction spectrum is measured by a setup consisting of a microwave horn antenna as the transmitter and a monopole antenna as the receiver. The size of the monopole antenna is 3.8 cm. Figure 3 shows the schematic view of the experimental setup. The 2D LHM slab has 10 layers along the incidence direction and 40 layers along the lateral direction. The horn antenna is on the negative side of the LHM structure with respect to its central axis. The source is 10 cm (1.5λ) from the first interface of the LHM slab. Full width at half-maximum of the beam at the interface is comparable to the wavelength and smaller than the size of the incident surface (5λ). The spatial intensity distribution along the second LHM–air interface is scanned in $\Delta x = 2.5$ mm steps. Initially, the EM wave is sent through the LHM sample with an incident angle of $\theta_i = 15°$.

Figure 4 shows the transmission spectrum as a function of the frequency and lateral position. The center of the outgoing Gaussian beam is shifted to the left side of the center of the incident Gaussian beam. This shift implies that the angle of refraction is negative; hence the refractive index, owing to Snell's law, becomes negative. The EM wave is evidently refracted at the negative side for frequencies in the range 3.84–3.89 GHz. Since at these frequencies better impedance matching is achieved, these experimental results cannot be evidence of any unexpected experimental artifacts such as reflection.

Figure 5 displays the refraction spectrum at 3.86 GHz, where the highest transmission and lowest reflection were observed. The center of the refracted Gaussian beam is −1.25 cm from the center of the incident Gaussian beam. We remind the reader that the incident field has a Gaussian beam profile centered at $x=0$ (not shown in the figure). One can easily find the refractive index value by applying Snell's law, where $n_{air} \sin \theta_i = n_{LHM} \sin \theta_r$. The angle of refraction can be defined in terms of the beam shift ($d_s$) and width of the LHM slab ($w_{LHM}$) as $\theta_r = \arctan(d_s / w_{LHM})$. The effective refractive index of the LHM is then calculated: $n_{\text{eff}} = -1.91$. We have previously performed wedge and phase shift experiments on the same structure.[13] The refractive index values obtained from wedge experiments ($n_{\text{eff}} = -1.95$) and phase shift experiments ($n_{\text{eff}} = -2.00$) are in quite good agreement with our experimental results.

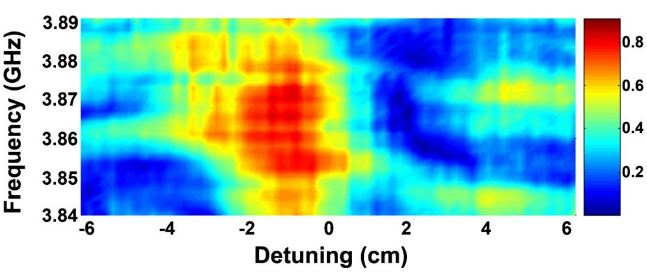


Fig. 4. (Color online) Transmission spectrum as a function of frequency and the lateral position at the LHM–air interface.

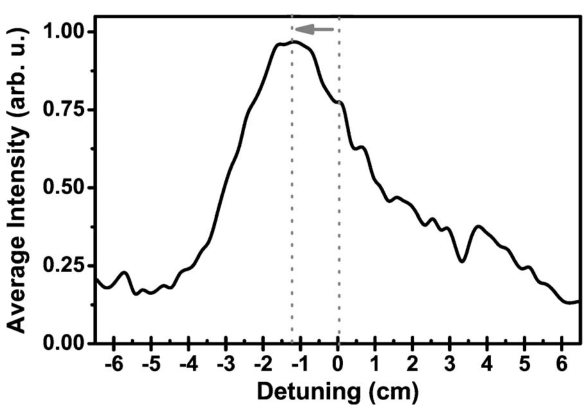


Fig. 5. Refraction spectrum at 3.86 GHz. The center of the Gaussian beam is shifted to the left, which means the refractive index is negative at this frequency.

**Table 1. Refractive Index Values for Different Incident Angles**

| Incident Angle | Peak Distance | Refractive Index |
|---|---|---|
| 10° | 7.5 mm | −2.11 |
| 15° | 12.5 mm | −1.91 |
| 20° | 15.0 mm | −2.10 |
| 30° | 25.0 mm | −1.89 |

We performed the same experiment for three more incident angles: $\theta_i = 10°$, 20°, and 30°. Table 1 shows the experimental results obtained from these measurements. The peak of the Gaussian beam is observed at 7.5, 15.0, and 25.0 mm from the center of the incident Gaussian beam, respectively. The effective refractive index values obtained from these measurements are −2.11 for $\theta_i = 10°$, −2.10 for $\theta_i = 20°$, and −1.89 for $\theta_i = 30°$. The agreement between the beam shift experiments for four different incident angles and also with the wedge and phase shift experiments is quite good. Since the index of refraction is determined by the material parameters effective $\epsilon$ and effective $\mu$, one in fact expects to obtain the same values for different angles of incidence. However, the situation is different in 2D photonic crystals, since refraction depends strongly on the wave vector.

## 3. CONCLUSION

In conclusion, we have studied the transmission and reflection characteristics of a two-dimensional LHM structure. A left-handed transmission band is observed in a frequency range where both effective permittivity and effective permeability are negative. The transmission peak value is quite high (−9.9 dB) for a structure made of metal. We have observed a very sharp dip in the reflection spectrum, which is due to impedance matching at the surface. The reflection is very low at 3.86 GHz with a dip value of −38 dB; therefore all of the incident EM waves can propagate inside the LHM structure. This is an important advance in metamaterial development, that when $\epsilon$ and $\mu$ are equal and both negative, then we obtain a well-matched, negative index material. The Gaussian beam shifting experiment is performed on the structure, and a negative refractive frequency region is observed. The effective index values obtained for four different angles of incidence are in good agreement.

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