

Imaging the near field

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Abstract. In an earlier paper we introduced the concept of the perfect lens which focuses both near and far electromagnetic fields, hence attaining perfect resolution. Here we consider refinements of the original prescription designed to overcome the limitations of imperfect materials. In particular we show that a multilayer stack of positive- and negative-refractive-index media is less sensitive to imperfections. It has the novel property of behaving like a fibre-optic bundle but one that acts on the near field, and not just the radiative component. The effects of retardation are included and minimized by making the slabs thinner. Absorption then dominates image resolution in the near field. The deleterious effects of absorption in the metal are reduced for thinner layers.

1. Introduction

Conventional optics is a highly developed subject but has limitations of resolution owing to the finite wavelength of light. It has been thought impossible to obtain images with details finer than this limit. Recently it has been shown that a 'perfect lens' is in principle possible and that arbitrarily fine details can be resolved in an image provided that the lens was constructed with sufficient precision. The prescription is simple: take a slab of material, of thickness d, and with electrical permittivity and magnetic permeability given by

$$\varepsilon = -1, \qquad \mu = -1. \tag{1}$$

Given that these conditions are realized, the slab will produce an image of any object with perfect resolution. The key to this remarkable behaviour is that the refractive index of the slab is

$$n = (\varepsilon \mu)^{1/2} = -1. \tag{2}$$

It was Veselago [1] in 1968 who first realized that negative values for ε and μ would result in a negative refractive index and he also pointed out that such a negative-refractive-index material would act as a lens (figure 1) but it took more than 30 years to realize the concept of negative refractive index at microwave frequencies [2–5].

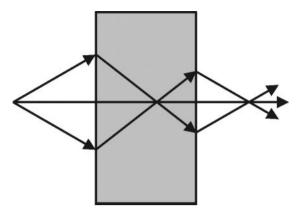


Figure 1. A negative-refractive-index medium bends light to a negative angle relative to the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time outside the medium.

It was only in recent times [6] that the lens's remarkable property of perfect resolution was noted. For the first time there is the possibility of manipulating the near field to form an image. The physics of a negative refractive index has caught the imagination of the physics community as evidenced by the publications in the past 2 years [4–15].

Although the conditions for a perfect lens are simple enough to specify, realizing them is in practice rather difficult. There are two main obstacles. Firstly, the condition of negative values for ε and μ also implies that these quantities depend very sensitively on frequency so that the ideal condition can only be realized at a single carefully selected frequency. Secondly, it is very important that absorption, which shows up as a positive imaginary component of ε or μ , is kept to a very small value. Resolution of the lens degrades rapidly with increasing absorption. It is the objective of this paper to explore how the effects of absorption can be minimized.

Let us probe a little deeper into the operation of the perfect lens. Any object is visible because it emits or scatters electromagnetic radiation. The problem of imaging is concerned with reproducing the electromagnetic field distribution of objects in a two-dimensional (2D) plane in the 2D image plane. The electromagnetic field in free space emitted or scattered by a 2D object (x–y plane) can be conveniently decomposed into the Fourier components k_x and k_y and polarization defined by σ :

$$E(x, y, z; t) = \sum_{k_x, k_y, \sigma} E_{\sigma}(k_x, k_y, k_z) \exp\left[i(k_x x + k_y y + k_z z - \omega t)\right], \tag{3}$$

where the source is assumed to be monochromatic at frequency ω , $k_x^2 + k_y^2 + k_z^2 = \omega^2/c^2$ and c is the speed of light in freespace. Obviously, when we move out of the object plane, the amplitude of each Fourier component changes (note the z dependence) and the image becomes blurred. The electromagnetic field consists of a radiative component of propagating modes with real k_z , and a near-field component of non-propagating modes with imaginary k_z whose amplitudes decay exponentially with distance from the source. Provided that k_z is real,

 $\omega^2/c^2 > k_x^2 + k_y^2$, it is only the phase that changes with z and a conventional lens is designed to correct for this phase change. The evanescent near-field modes are the high-frequency Fourier components describing the finest details in the object and to restore their amplitudes in the image plane requires amplification, which is of course beyond the power of a conventional lens and hence the limitations to resolution (figure 2).

Thus the perfect lens performs the dual function of correcting the phase of the radiative components as well as amplifying the near-field components, bringing them both together to make a perfect image, thereby eliminating the diffraction limit on the image resolution. In general the conditions under which this perfect imaging occurs are

$$\varepsilon_{-} = -\varepsilon_{+}, \qquad \mu_{-} = -\mu_{+}, \tag{4}$$

where ε_{-} and μ_{-} are the dielectric permittivity and magnetic permeability respectively of the negative refractive material slab, and ε_{+} and μ_{+} are the dielectric permittivity and magnetic permeability respectively of the surrounding medium.

An important simplification of these conditions can occur in the case when *all* length scales are much less than the wavelength of light. Under these circumstances, electric and magnetic fields decouple: the P-polarized component of light becomes mainly electric in nature, and the S-polarized component mainly magnetic. Therefore in the case of P-polarized light we need only require that $\varepsilon=-1$, and the value of μ is almost irrelevant. This is a welcome relaxation of the requirements especially at optical frequencies where many materials have a negative values for ε but show no magnetic activity. We shall concentrate our investigations on these extreme near field conditions and confine our attention to P-polarized light.

In section 2, we investigate the properties of a layered structure consisting of extremely thin slabs of silver and show that layered structures are less susceptible to the degrading effects of absorption than are single-element lenses. In section 3,

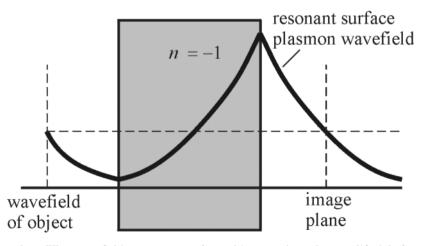


Figure 2. The near-field component of an object needs to be amplified before it can make its contribution to an image. This can be done by resonantly exciting surface plasmons on the right-hand surface. The condition n = -1 ensures existence of surface plasmon modes at the operating frequency.

we present some detailed calculations of how the multilayer lens transmits the individual Fourier components of the image.

2. The layered perfect lens, an unusual effective medium

Reference to figure 2 shows that extremely large amplitudes of the electric field occur within the lens when the near field is being amplified. This is especially true for the high-frequency Fourier components which give the highest resolution to the image. Unless the lens is very close to the ideal lossless structure, these large fields will result in dissipation which will stop the amplifying effect. However, there is a way to restructure the lens to ameliorate the effects of dissipation. We observe that in the ideal lossless case we can perfectly well divide the lens into separate layers, each making its contribution to the amplification process (Shamomina et al. have made a similar observation and Zhang and Fu [15] have considered a similar system [15]). Provided that the total length of vacuum between the object and image is equal to the total length of lens material, the lens will still work and produce a perfect image. However, this subdivision of the lens makes a large difference as to how the lens performs when it is less than ideal and absorption is present. The point is that, by distribution of the amplification, the fields never grow to the extreme values that they do when the lens is a single slab and therefore the dissipation will be much less. Figure 3 illustrates this point.

First let us estimate the resolution of a lens constituted as a single slab. According to our original calculations [6] in the near-field limit, the transmission coefficient through the lens for each Fourier component is

$$\frac{\exp\left[-2(k_x^2 + k_y^2)^{1/2} d\right]}{\frac{1}{4}(\varepsilon_-'')^2 + \exp\left[-2(k_x^2 + k_y^2 d)^{1/2}\right]},\tag{5}$$

where

$$\varepsilon_{-} = \varepsilon'_{-} + i\varepsilon''_{-}$$
.

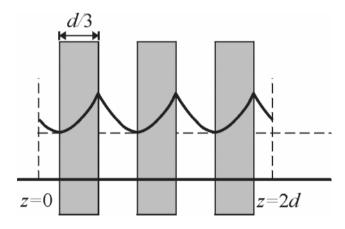


Figure 3. Schematic diagram of the field distribution for an incident evanescent wave on a layered perfect lens, when the original lens is cut into three pieces placed symmetrically between object and image.

Obviously when

$$\left(k_x^2 + k_y^2\right)^{1/2} d < \ln\left(\frac{\varepsilon_-''}{2}\right),\tag{6}$$

the lens' power to amplify begins to fall away. Fourier components of higher spatial frequency do not contribute and hence the resolution is limited to

$$\Delta = \frac{2\pi d}{\ln\left(\varepsilon''_{-}/2\right)}.\tag{7}$$

The easiest way to investigate the properties of a layered system is to recognize that, provided that the slices are thin enough, it will behave as an effective anisotropic medium whose properties we calculate as follows (figure 4). Applying a uniform displacement field D perpendicular to the slices gives electric fields of $\varepsilon_0^{-1}\varepsilon_+^{-1}D$ and $\varepsilon_0^{-1}\varepsilon_-^{-1}D$ in the positive dielectric medium and in the negative material of the lens respectively. Therefore the average electric field is given by

$$\langle E \rangle = \frac{1}{2} \left(\varepsilon_0^{-1} \varepsilon_+^{-1} D + \varepsilon_0^{-1} \varepsilon_-^{-1} D \right) = \varepsilon_0^{-1} \varepsilon_z^{-1} D, \tag{8}$$

where

$$\varepsilon_z^{-1} = \frac{1}{2} (\varepsilon_+^{-1} + \varepsilon_-^{-1}), \tag{9}$$

is the effective dielectric function for fields acting along the z axis. By considering an electric field along the x axis we arrive at

$$\varepsilon_x = \frac{1}{2}(\varepsilon_+ + \varepsilon_-),\tag{10}$$

where ε_x is the effective dielectric function for fields acting along the x axis. We have assumed for simplicity that the thicknesses of all material components are the same, but it is also possible to have unequal thicknesses. Now under the perfect lens conditions, $\varepsilon_- = -\varepsilon_+$, we have

$$\varepsilon_z \to \infty, \quad \varepsilon_x \to 0.$$
 (11)

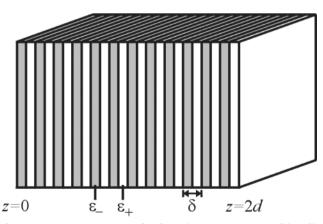


Figure 4. In the extreme we can cut the lens into very many thin slices so that we reduce the effects of absorption as much as possible. In the limit of infinitesimal slices the ensemble can be treated as an effective medium with an anisotropic dielectric function.

Thus the stack of alternating extremely thin layers of negative- and positive-refractive-index media in the limiting case when the layer thickness goes to zero behaves as a highly anisotropic medium.

Radiation propagates in an anisotropic medium with the following dispersion:

$$\frac{k_x^2 + k_y^2}{\varepsilon_x} + \frac{k_z^2}{\varepsilon_y} = \frac{\omega^2}{c^2},\tag{12}$$

and hence for the perfect lens conditions it is always true that

$$k_z = 0. (13)$$

Each Fourier component of the image passes through this unusual medium without change in phase or attenuation. It is as if the front and back surfaces of the medium were in immediate contact.

Here we have a close analogy with an optical fibre bundle where each fibre corresponds to a pixel and copies the amplitude of the object pixels to the image pixels without attenuation and with the same phase change for each pixel, preserving optical coherence. Our layered system performs exactly the same function with the refinement that in principle the pixels are infinitely small, and the phase change is zero. In figure 5 we illustrate this point with an equivalent system: an array of infinitely conducting wires embedded in a medium where $\varepsilon = 0$. In the latter case it is more obvious that an image propagates through the system without distortion. Indeed in the trivial zero frequency limit the system simply connects object to image point by point.

Returning to our point that the layered system reduces the effect of absorption, we estimate the transmission for P-polarized light through such a system in the near-field limit as

$$T_{\rm P} \approx \frac{1}{\cos\left[(\mathrm{i}/2)\varepsilon_{-}^{"}k_{x}2d\right] + \frac{1}{2}(\varepsilon_{+} + \varepsilon_{+}^{-1})\,\sin\left[(\mathrm{i}/2)\varepsilon_{-}^{"}k_{x}2d\right]}.\tag{14}$$

Evidently, for small values of k_x , the transmission coefficient is unity and these Fourier components contribute perfectly to the image but, for large values of k_x , transmission is reduced. We estimate the resolution limit to be

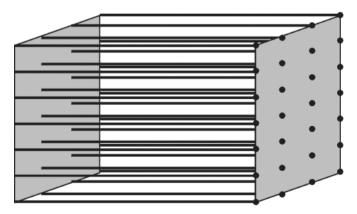


Figure 5. An array of very closely spaced infinitely conducting wires, embedded in a medium where $\varepsilon = 0$, behaves in the same manner as the stack of very thin sheets shown in figure 4.

$$\Delta \simeq \frac{2\pi}{k_{\text{max}}} = 2\pi (\frac{1}{2}\varepsilon''_{-}2d) = 2\pi\varepsilon''_{-}d. \tag{15}$$

Therefore the smallest detail resolved by the lens decreases linearly with decreasing absorption ($\varepsilon_{-}^{"}$). In contrast the original single slab of lens had a much slower improvement in resolution, being inversely as $\ln(\varepsilon_{-}^{"})$. Thus it appears to be the case that two lenses are better than one but many lenses are the best of all.

3. Image simulations for a multilayer stack

In the previous section we gave some qualitative arguments as to the properties of metal-dielectric multilayer stacks and is clear that for P-polarized light in the quasistatic limit this structure would behave as a near-perfect 'fibre optic bundle'. In the electrostatic (magnetostatic) limit of large $k_x \sim k_z \sim q_z$, there is no effect of changing $\mu(\varepsilon)$ for the P(S) polarization. The deviation from the quasistatic limit caused by the non-zero frequency of the electromagnetic wave would, however, not allow this decoupling. When the effects of retardation are included, a mismatch in the ε and μ from the perfect-lens conditions would always limit the image resolution and also leads to large transmission resonances associated with the excitation of coupled surface modes that could introduce artefacts into the image [16]. For the negative dielectric (silver) lens, the magnetic permeability $\mu=1$ everywhere, and this is a large deviation from the perfect lens conditions. The dispersion of these coupled slab plasmon polaritons and their effects on the image transfer has been extensively studied in [13].

Essentially, for a single slab of negative dielectric material which satisfies the conditions for the existence of a surface plasmon on both the interfaces, the two surface plasmon states hybridize to give an antisymmetric and a symmetric state, whose frequencies are detuned away from that of a single uncoupled surface state. The transmission as a function of the transverse wave-vector remains reasonably close to unity up to the resonant wave-vector for the coupled plasmon state, after which it decays exponentially with larger wave-vectors. The secret for better image resolution is to obtain a flat transmission coefficient for as large a range of wave-vectors as possible. This is possible by using a thinner slab in which case the transmission resonance corresponding to a coupled slab mode occurs at a much larger k_x . For the transfer of the image over useful distances, we would then have to resort to a layered system of very thin slabs of alternating positive and negative media.

Let us now consider a layered system consisting of thin slabs of silver (negative dielectric constant ε_-) and any other positive dielectric medium (ε_+). Since the dielectric constant of silver is dispersive†, we can choose the frequency ω of the electromagnetic radiation so as to satisfy the perfect lens condition at the interfaces between the media ($\varepsilon_-(\omega) = -\varepsilon_+(\omega)$). We use the transfer matrix method [17] to compute the transmission through the layered medium as a function of the transverse wave-vector at a frequency at which the perfect lens condition is

†An empirical form for the dielectric constant of silver in the visible region of the spectrum is $\varepsilon_-(\omega) = 5.7 - 9.0^2 (\hbar \omega)^{-2} + \mathrm{i}0.4$ ($\hbar \omega$ in electron volts). The imaginary part can be taken to be reasonably constant in this frequency range. We note, however, that the imaginary part of the effective dielectric constant can be higher for very thin slabs owing to enhanced effects of surface scattering of the electrons.

satisfied. We shall denote by N the number of slabs with negative dielectric constant in the alternating structure, each period consisting of a negative and positive slab as shown in figure 4. Now the total length of the system is $2d = N\delta$, where δ is the period of the multilayer stack (the negative and positive slabs being of equal thickness $\delta/2$). Note that the total thicknesses of positive and negative dielectric media between the object plane to the image plane are also equal.

The transmission across the multilayer system is shown in figure 6, where the thicknesses of the individual slabs is kept constant, but the number of layers is

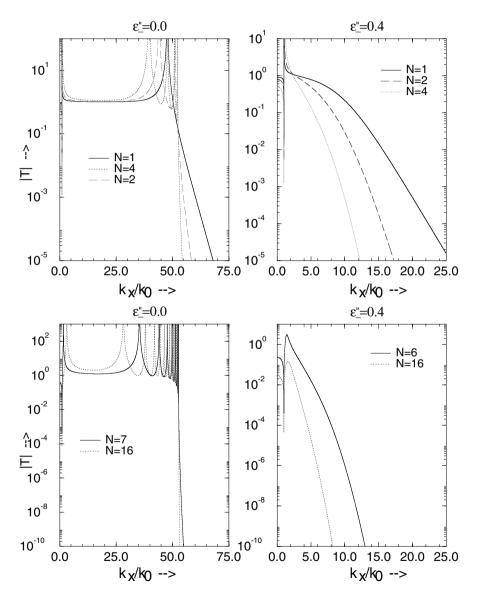


Figure 6. The transmission coefficient as a function of k_x , for the metal-dielectric multilayer stack. $\varepsilon_+ = 1.0$, $\varepsilon_-' = -\varepsilon_+$ and $\delta = 20$ nm. The graphs on the left are for a hypothetical lossless medium and on the right for silver with $\varepsilon_-'' = 0.4$. The layer thickness is kept constant and the number of layers increased.

increased, thereby increasing the total length of the system. We obtain divergences in the transmission at wave-vectors corresponding to the coupled plasmon resonances. The number of the resonances increases with increasing number of layers, corresponding to the number of surface modes at the interfaces. For the system with the (hypothetical) lossless negative media, one notes that, as we increase the number of layers, the transmission coefficient is almost constant and close to unity with increasing k_x , until it passes through the set of resonances and decays exponentially beyond. The range of k_x for which the transfer function is constant is independent of the total number of layers and depends only on the thickness of the individual layers, which sets the coupling strength for the plasmon states at the interfaces. In the presence of absorption in the negative medium, however, the decay is extremely fast for the system with larger N simply as a consequence of the larger amount of absorptive medium present. Also note that the absorption removes all the divergences in the transmission. As noted by us in earlier publications, the absorption is actually vital in this system to prevent the resonant divergences which would otherwise create artefacts that dominate the image.

Next we keep the total length of the stack fixed and change the number of layers. In the lossless case, the range of k_x for which there is effective amplification of the evanescent waves simply increases with reducing layer thickness as can be seen in figure 7. Of course, the number of transmission resonances which depend on the number of surface states increases with increasing number of layers. With absorptive material, however, the transmission decays faster with k_x for larger k_x in the case of the thicker slabs (10 nm) than in the case of the thinner slabs (5 nm). This reconfirms our analytical result that the effects of absorption would be less

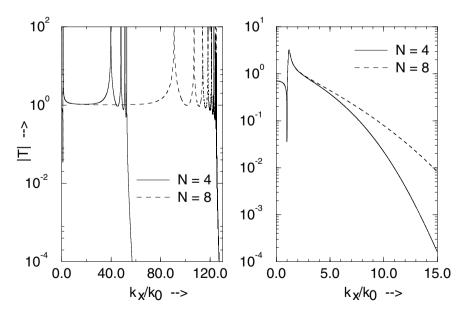


Figure 7. The transmission coefficient for two layered stacks of equal total thicknesses $2d=80\,\mathrm{nm}$, but $\delta/2=10\,\mathrm{nm}$ (N=4) and $\delta/2=5\,\mathrm{nm}$ (N=8) in the two cases. The graph on the left is for a hypothetical lossless medium, and on the right for silver.

deleterious for the image resolution in the case of thinner layers. Note that the total amounts of absorptive material in this case are the same in both the cases.

In any case, the absorption in the negative dielectric (metal) appears to set the ultimate limit on the image resolution in this case of the layered medium. We have noted earlier in [13] that the effects of absorption could be minimized by using a large dielectric constant, GaAs, say ($\varepsilon_+ = 12$), for the positive medium and tuning to the appropriate frequency where the perfect lens condition $\varepsilon'_{-} = -\varepsilon_{+}$ is satisfied for the real part of the dielectric constant ε' of the metal. In the case of silver, the imaginary part of the permittivity or the absorption is reasonably constant (about 0.4) over the frequency range of interest. Hence, it is immediately seen that the fractional deviation from the perfect lens condition in the imaginary part is smaller when the real part of the permittivity is large and hence the amplification of the evanescent waves becomes more effective. Now we show the transmission obtained across a multilayer stack where $\varepsilon_+ = 12$ and $\varepsilon_- = -12 + i0.4$, corresponding to alternating slabs of silver and GaAs, in figure 8. We must first note that the wavelengths of light at which the perfect lens condition for the permittivity of silver is satisfied are different in the two cases. Using the empirical formula for the dispersion of silver, we obtain $\varepsilon'_{-}=-1$ at 356 nm and $\varepsilon'_{-}=-12$ at 578 nm. In figure 8, for the lossless system, the transmission resonances appear to occur at higher values of k_x/k_0 for the high-index system, but it must be realized that $k_0 = 2\pi/\lambda$ is smaller in this case and the corresponding image resolution would actually be lower. However, when we compare the transmission with absorption included, the beneficial effects of using the larger value of the dielectric constant become obvious. The transmission coefficient indeed decays much more slowly with increasing k_x in this case. Also note that we have taken the source to be in air and the image to be formed inside the high-index dielectric medium.

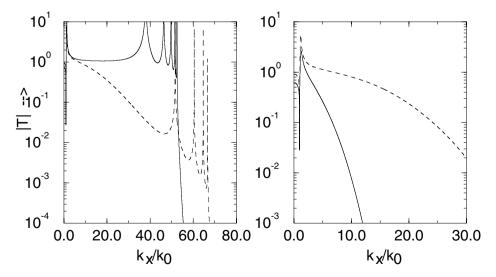


Figure 8. The transmission coefficient for two layered stacks of equal total thicknesses $2d=100\,\mathrm{nm}$ and $\delta/2=10\,\mathrm{nm}$. The graph on the left is for a hypothetical lossless medium, and on the right for silver $\epsilon''_-=0.4$. On the right the solid curve is for $\epsilon'_-=-1$ and the broken curve for $\epsilon'_-=-12$.

Finally, we compare the images formed by a single-slab lens and a layered lens, silver being the lens material in both cases. Assuming invariance along the y axis (line sources) for simplicity, if $E_s(k_x)$ is the Fourier spectrum of the field distribution on the object plane and $T_L(k_x)$ is the transmission function for transfer to the image plane, then the field distribution at the image plane is given by

$$E_{i}(x) = \sum_{k_{x}} \exp(+ik_{x}x) T_{L}(k_{x}) E_{s}(k_{x}).$$
 (16)

We show in figure 9 the images of two slits of $30\,\mathrm{nm}$ width and a peak-to-peak separation of $90\,\mathrm{nm}$ obtained by using a single slab of silver as the lens and a layered medium of alternating layers of silver and a positive dielectric medium as the lens. The total distance from the object plane to the image plane in both cases is $2d=80\,\mathrm{nm}$. The images of the slits in the case of the single-slab lens are just resolved, whereas the images of the slits are well separated and clearly resolved in the case of the layered lens. The enhancement in the image resolution for the layered lens is obvious from the figure. The bump seen in between the slits is an artefact arising because the transmission function is not exactly a constant for all wave-vectors.

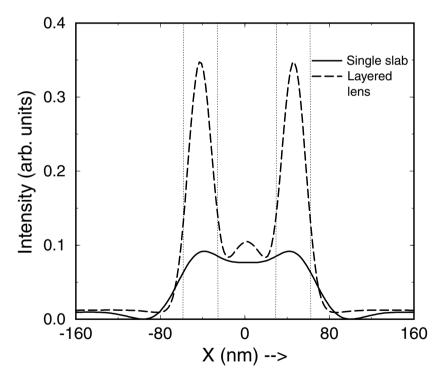


Figure 9. The electromagnetic field intensity at the image plane for an object consisting of two slits of 30 nm width and a peak-to-peak separation of 90 nm obtained by using, firstly a single slab of silver of thickness 40 nm and, secondly a layered stack of alternating positive and negative dielectric layers of $\delta/2 = 5$ nm layer thicknesses and where the number of layers is N = 8. The object plane to image plane distance is 2d = 80 nm in both the cases.

4. Conclusions

We have elaborated the design of the perfect lens by considering a multilayer stack and shown that this has advantages over the original configuration of a single slab of material. In particular the effects of absorption are much reduced by division into multilayers. The limiting case of infinitesimal multilayers was also considered and shown to be equivalent to an effective medium through which the image propagates without distortion as if it were conveyed by an array of very fine infinitely conducting wires. We went on to make a detailed analysis of how imperfections in the lens affects the image quality. The effects of retardation and the coupled slab plasmon resonances can be minimized by considering very thin layers of 5–10 nm thickness. The effects of absorption then dominate the image transfer but are less deleterious when the individual layer thicknesses are smaller. The effects of absorption can also be minimized by using materials with higher dielectric constants, and tuning the frequency of the radiation to meet the perfect lens conditions.

Acknowledgments

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