

# Theoretical study of left-handed behavior of composite metamaterials

R.S. Penciu<sup>a,b,\*</sup>, M. Kafesaki<sup>a,b</sup>, T.F. Gundogdu<sup>a,b</sup>,  
E.N. Economou<sup>a,c</sup>, C.M. Soukoulis<sup>a,b,d</sup>

<sup>a</sup> *Foundation for Research and Technology Hellas, Institute of Electronic Structure and Laser, FORTH, Vassilika Vouton, 71110 Heraklion, Greece*

<sup>b</sup> *Department of Materials Science and Technology, University of Crete, Greece*

<sup>c</sup> *Department of Physics, University of Crete, Greece*

<sup>d</sup> *Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, IW 50011, USA*

Received 7 October 2005; received in revised form 3 November 2005; accepted 5 November 2005

Available online 1 December 2005

## Abstract

We investigate numerically the transmission properties of 1D and 2D composite metamaterials (CMM) consisting of periodically arrangements of circular split-ring resonators (SRR) and wires. The theoretical methods used are the commercially available code Microwave Studio for transmission calculations and a retrieval procedure which gives the effective electric permittivity,  $\epsilon$ , and magnetic permeability,  $\mu$ , of the system. Our theoretical results are in good agreement with experimental data. © 2005 Elsevier B.V. All rights reserved.

PACS: 41.20.Jb; 73.20.Mf; 78.20.Ci

Keywords: Left-handed materials; Negative refraction; Electromagnetic waves

## 1. Introduction

In the last few years there have been many studies concerning metamaterials with negative refractive index. The discovery of these metamaterials emerged from the pioneering conceptual idea of Veselago [1] and the proposal for the structure by Pendry et al. [2,3].

By employing several theoretical methods, we studied the behavior of structures made from wires and split-ring resonators of various geometries at a wide range of operating frequencies. The transfer matrix method (TMM) is a suitable method for pure theoretical studies. Thus, using this method, we can assess the SRR

electric response [4,5], the electric field coupling to the magnetic resonance of SRR [6], the dependence of transmission on parameters of the system, material properties, and orientation [7], more symmetric, multigap SRR designs proposed for 3D left-handed metamaterials [8], the effect of periodicity on the effective medium approximation [9], the limit of the SRR magnetic response at optical frequencies [10], etc. This method is not suitable though for the description of a real system, because a large number of mesh-cells may be required for certain SRR and wire geometries which makes the calculation intractable, mostly in the time scale. In this case, we use the finite integration technique employed through Microwave Studio commercial software for transmission calculations together with a retrieval procedure [11], which extracts the effective electric permittivity,  $\epsilon$ , and magnetic permeability,  $\mu$ ,

\* Corresponding author. Tel.: +30 2810 391337;  
fax: +30 2810 391305.

E-mail address: [raluca@iesl.forth.gr](mailto:raluca@iesl.forth.gr) (R.S. Penciu).

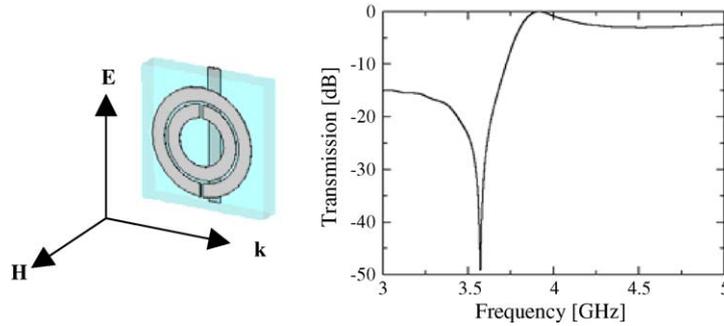


Fig. 1. The right panel shows the calculated transmission spectrum for one unit cell in the propagation direction. The polarization and propagation directions are shown in the left panel; the propagation direction is parallel to the SRR plane and the electric field,  $\mathbf{E}$ , is parallel to the wire.

by inverting the reflection-transmission results, considering our metamaterial as a homogeneous effective medium.

## 2. 1D metamaterial with left-handed behavior at $\sim 4$ GHz

In this section, we theoretically study the CMM structure experimentally measured by Aydin et al. [12]. This is a 1D periodic structure made of one circular SRR and one continuous wire per unit cell. The geometric parameters of the system are the width of the SRR and wire, 0.9 mm; the SRR gap and the distance between the two SRR rings, 0.2 mm; the inner radius of the inner ring, 1.6 mm; and the SRR and wire thickness, 30  $\mu\text{m}$ . The dielectric board has a 1.6 mm thickness

and, in order for the theoretical transmission left-handed (LH) peak to be centered at the same frequency as the experimental peak, we choose the dielectric constant of the board,  $\epsilon = 2.6$ . The unit cell dimensions are  $a_{\mathbf{k}} = a_{\mathbf{E}} = 8.8$  mm and  $a_{\mathbf{H}} = 6.5$  mm, where the polarization and the propagation directions are shown in the left panel of Fig. 1. The right panel of Fig. 1 presents the calculated transmission spectrum for one unit cell in the propagation direction and periodic conditions in the other two directions.

Aydin et al. [12] compared the simulations with the experimental transmission spectra through 5 unit cells in the propagation direction and they obtained a very good agreement. Then they experimentally checked that the transmission peak is LH by comparing these results with results for closed SRR and wires samples.

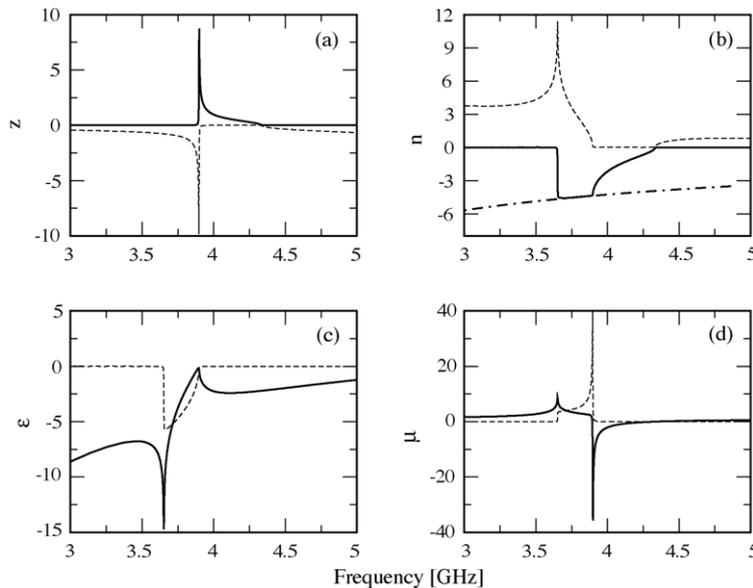


Fig. 2. The retrieved real (solid line) and imaginary (dashed line) parts of the effective impedance,  $z$  (a), the refractive index,  $n$  (b), the electric permittivity,  $\epsilon$  (c), and the magnetic permeability,  $\mu$  (d), for the structure presented in Fig. 1. The dot-dashed line shows the minimum value of the refractive index in the first Brillouin zone.

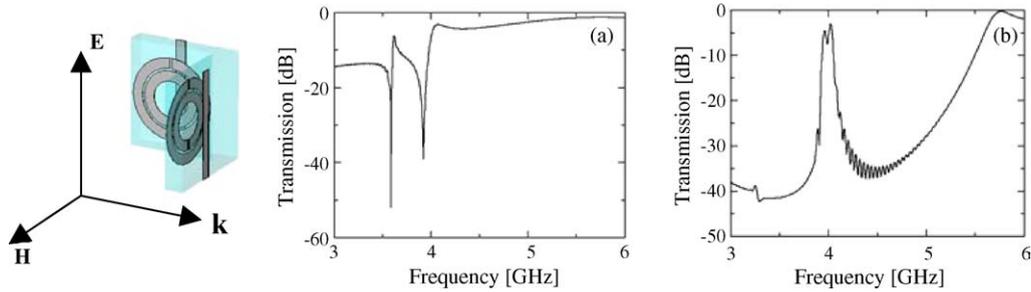


Fig. 3. Transmission spectra for one (a) and five (b) unit cells in the propagation direction. The polarization and propagation directions are shown in the left panel.

Here, we will demonstrate the LH character by calculating the impedance,  $z$ , the refractive index,  $n$ , the electric permittivity,  $\varepsilon$ , and the magnetic permeability,  $\mu$  with the well developed retrieval method.

Fig. 2 shows the negative values of the real parts of  $n$ ,  $\varepsilon$ , and  $\mu$ . The real part of  $n$  is cut by the minimum value in the first Brillouin zone. The periodicity influences also the magnetic resonance in  $\mu$ . The real part of  $\varepsilon$  is negative for all frequencies, which confirms that we are below the “plasma frequency” of the CMM.

### 3. 2D metamaterial with left-handed behavior at $\sim 4$ GHz

We consider the case of a 2D CMM with LH behavior. The experimental results for this structure are presented in ref. [13]. The materials and the SRR and

wire geometries are the same as in the previous 1D case, while the unit cell is  $a_{\mathbf{k}} = a_{\mathbf{E}} = a_{\mathbf{H}} = 9.3$  mm.

As Fig. 3(a) shows, the magnetic resonance of the single SRR splits in the presence of the second SRR in the unit cell. When more unit cells are added in the propagation direction, only the width of the LH transmission at higher frequencies increases (see Fig. 3(b)), the lower frequency peak being too weak to be experimentally detected. In this case, the board dielectric constant value which provides theoretical results in agreement with the experimental data is  $\varepsilon = 3.6$ . This is a little different than the values stated in the experiments [12]. This might be due to the fact that the dielectric boards may not be uniform. Using the retrieval procedure, we show (see Fig. 4) the splitting of the resonance in  $\mu$  and the negative real part of  $\varepsilon$ . The refractive index has a negative value for two frequency

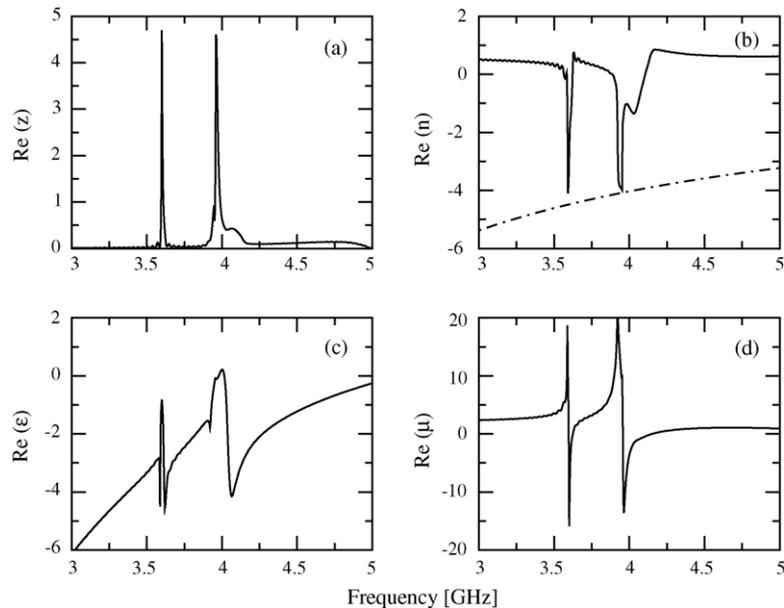


Fig. 4. The retrieved real part of the effective impedance,  $z$  (a), the refractive index,  $n$  (b), the electric permittivity,  $\varepsilon$  (c), and the magnetic permeability,  $\mu$  (d), for the structure presented in Fig. 3, and for one unit cell in the propagation direction. The dot-dashed line shows the minimum value of the refractive index in the first Brillouin zone.

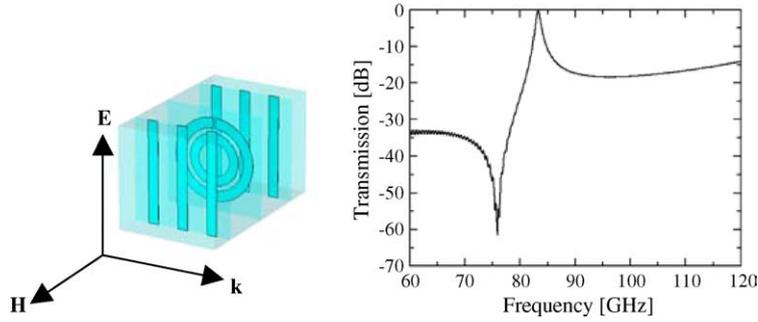


Fig. 5. The right panel shows the transmission spectrum for one unit cell in the propagation direction. The polarization and propagation directions are shown in the left panel; the propagation direction is parallel to the SRR plane and the electric field,  $E$ , is parallel to the wires.

ranges, the range at lower frequencies being narrower than the one at higher frequencies.

#### 4. 1D metamaterial with left-handed behavior at ~100 GHz

The last structure that will be examined is a 1D CMM, which exhibits LH transmission peak at 100 GHz. In this case, two layers of three continuous wires are needed per unit cell (see the left panel of Fig. 5) in order to obtain the plasma frequency of the CMM consisting of SRRs and wires,  $\omega_p'$ , higher than the magnetic frequency of the SRR-only system. The dimensions of the components are inner ring inner

radius,  $43 \mu\text{m}$ ; inner ring outer radius,  $67.2 \mu\text{m}$ ; outer ring inner radius,  $80.7 \mu\text{m}$ ; outer ring outer radius,  $107.5 \mu\text{m}$ ; split ring gap,  $7.2 \mu\text{m}$ ; wire width,  $26.9 \mu\text{m}$ ; wire separation,  $53.7 \mu\text{m}$ ; SRR and wire thickness,  $0.5 \mu\text{m}$ ; and dielectric board thickness,  $150 \mu\text{m}$ . The periodicity in the SRR plane is  $a_k = a_E = 262.7 \mu\text{m}$ . The dielectric constant for the substrate is  $\epsilon = 6.2$ .

Similarly to Figs. 2 and 4, in Fig. 6 one can observe the negative  $n$ ,  $\epsilon$ , and  $\mu$ , and the influence of the periodicity. The frequency domain for which  $\text{Re}(n) < 0$  extends from 93 to 104 GHz, while the experimental CMM structure [14] exhibits a transmission band from 98 to 104 GHz. The transmission at the lower frequencies of the  $\text{Re}(n) < 0$  domain is suppressed, due to the high  $\text{Im}(n)$ .

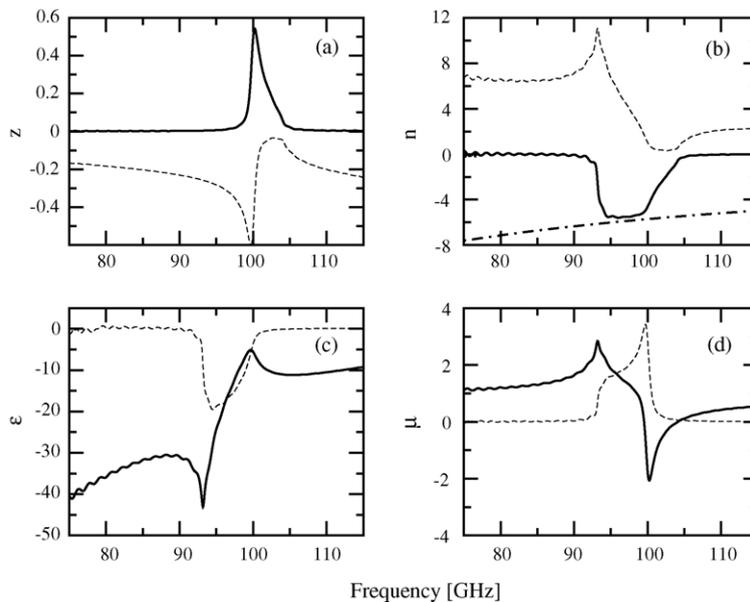


Fig. 6. The retrieved real (thick) and imaginary (thin) parts of the effective impedance,  $z$  (a), the refractive index,  $n$  (b), the electric permittivity,  $\epsilon$  (c), and the magnetic permeability,  $\mu$  (d), for the structure presented in Fig. 5. The dot-dashed line shows the minimum value of the refractive index in the first Brillouin zone.

## 5. Conclusion

By employing the Microwave Studio code and the retrieval procedure, we obtained results in good agreement with the experimental data. This shows that we can successfully use these theoretical methods to predict designs for CMM with LH behavior fabrication. However, there are discrepancies between theory and experiment, such as differences in the transmission and width of the LH peak, due to the fact that the finite thickness of the metallic components, experimental disorder and misalignment, and fabrication-based non-uniformity of SRRs and wires are not considered in our theoretical methods.

## Acknowledgements

This work was partially supported by Ames Laboratory (Contract number W-7405-Eng-82). Financial support by EU-projects DALHM, Metamorphose, Phoremest, and by the Greek Ministry of Education (through PYTHAGORAS project) are also acknowledged.

## References

- [1] V.G. Veselago, *Ups. Fiz. Nauk* 92 (1967) 517; V.G. Veselago, *Engl. Trans. Sov. Phys. Ups.* 10 (1968) 509.
- [2] J.B. Pendry, A.J. Holden, W.J. Stewart, I. Youngs, *Phys. Rev. Lett.* 76 (1996) 4773; J.B. Pendry, A.J. Holden, W.J. Stewart, I. Youngs, *J. Phys. Condens. Matter.* 10 (1998) 4785.
- [3] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Stewart, *IEEE Trans. Microwave Theory Tech.* 47 (1999) 2057.
- [4] Th. Koschny, M. Kafesaki, E.N. Economou, C.M. Soukoulis, *Phys. Rev. Lett.* 93 (2004) 107402.
- [5] N. Katsarakis, Th. Koschny, M. Kafesaki, E.N. Economou, E. Ozbay, C.M. Soukoulis, *Phys. Rev. B* 70 (2004) 201101.
- [6] N. Katsarakis, Th. Koschny, M. Kafesaki, E.N. Economou, C.M. Soukoulis, *Appl. Phys. Lett.* 84 (2004) 2943.
- [7] M. Kafesaki, Th. Koschny, R.S. Penciu, T.F. Gundogdu, E.N. Economou, C.M. Soukoulis, *J. Opt. A: Pure Appl. Opt.* 7 (2005) S12.
- [8] Th. Koschny, L. Zang, C.M. Soukoulis, *Phys. Rev. B* 71 (2005) 121103.
- [9] Th. Koschny, P. Markos, E.N. Economou, D.R. Smith, D.C. Vier, C.M. Soukoulis, *Phys. Rev. B* 71 (2005) 245105.
- [10] J. Zhou, Th. Koschny, M. Kafesaki, E.N. Economou, J.B. Pendry, C.M. Soukoulis, *Phys. Rev. Lett.* 95 (2005) 223902; N. Katsarakis, G. Konstantinidis, A. Kostopoulos, R.S. Penciu, T.F. Gundogdu, M. Kafesaki, Th. Koschny, E.N. Economou, C.M. Soukoulis, *Opt. Lett.* 30 (2005) 1348.
- [11] D.R. Smith, S. Schultz, P. Markos, C.M. Soukoulis, *Phys. Rev. B* 65 (2002) 195104.
- [12] K. Aydin, K. Guven, M. Kafesaki, L. Zhang, C.M. Soukoulis, E. Ozbay, *Opt. Lett.* 29 (2004) 2623.
- [13] K. Aydin, K. Guven, C.M. Soukoulis, E. Ozbay, *Appl. Phys. Lett.* 86 (2005) 124102.
- [14] M. Gokkavas, K. Guven, I. Bulu, K. Aydin, R.S. Penciu, M. Kafesaki, C.M. Soukoulis, E. Ozbay, *Phys. Rev. Lett.*, submitted for publication.